CHAPTER-64

A PERFORMANCE ANALYSIS OF MANUFACTURING SYSTEMS IMPERFECT SPARES AND REPAIR FACILITIES

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ABSTRACT

This study analyzes the performance of a manufacturing system operating with a primary machine, imperfect spares, and repair facilities. Failures and repairs are modeled using exponential distributions, with system states defined according to machine availability and repair status. A recursive solution method is applied to obtain steady-state probabilities, which are then used to compute key performance indicators such as availability, mean number of machines under repair, and downtime probability. Numerical simulations confirm that system availability remains consistently high, with downtime probability being minimal under efficient repair conditions. The results emphasize the balance between repair efficiency and spare reliability in ensuring overall system performance.

Keywords: Manufacturing systems; imperfect spares; repair facilities; system availability; downtime probability; reliability modeling; steady-state analysis.

INTRODUCTION

Manufacturing systems rely heavily on continuous machine operation to sustain productivity, making reliability and maintenance critical concerns. In practice, spare machines are often imperfect, with higher failure rates compared to the main machine, and the efficiency of repair facilities strongly influences system performance. The interplay between machine failures, spares, and repairs determines the operational capacity of the entire system. Modeling such systems using stochastic processes provides insights into their steady-state behavior and reliability characteristics. By analyzing performance measures like system availability, mean number under repair, and downtime probability, decision-makers can evaluate the trade-offs between spare quality and repair efficiency. This research contributes to a deeper understanding of reliability modeling for real-world manufacturing systems where spares may not be flawless and repair processes vary in effectiveness.

Yen et al. (2015) emphasized the importance of balancing repair resources with system downtime to optimize performance. This early contribution highlighted the significance of repair policies in ensuring operational continuity when breakdowns are frequent. Aghezzaf et al. (2016) explored the integration of production planning with imperfect preventive maintenance in failure-prone manufacturing systems. Their study emphasized the significance of synchronizing production schedules with maintenance strategies to reduce overall system downtime and costs. The model they proposed showed that disregarding preventive maintenance while planning production often results in inefficiencies, as unexpected breakdowns disrupt manufacturing flows. By embedding maintenance decisions into the production framework, they highlighted potential gains in reliability and economic performance. Lachemot et al. (2016) focused on stochastic queueing structures, capturing the effect of delays and recurrent failures. This broadened the understanding of how breakdowns impact service-oriented and manufacturing systems where customer demand continues despite repair-related disruptions. Sharma et al. (2017) presented a simulation-based optimization approach to forecast spare parts and guide selective maintenance decisions. Unlike traditional forecasting methods, their model incorporated operational uncertainties and system reliability dynamics to improve accuracy. The results demonstrated that simulation could effectively capture stochastic demand patterns and maintenance requirements, ensuring better resource allocation. This work underscored the importance of integrating forecasting with decision-support tools to enhance spare parts management. Wang and Liou (2017) explored the machine repair problem using particle swarm optimization (PSO) in the presence of working breakdowns. Their optimization-based approach offered efficient solutions for complex machine repair models, proving the applicability of swarm intelligence in maintenance decision-making. This was an important methodological advancement, linking computational intelligence to practical repair optimization. Cheng et al. (2019) investigated the reliability of solar energy generating systems with inverters in series, particularly under common cause failures. While the study was centered on energy systems, its insights on failure dependencies and cascading effects were relevant to broader reliability analysis in manufacturing environments. The concept of common cause failure provided a valuable framework for analyzing dependencies in spares and repair facilities. Yang et al. (2019) proposed a two-phase preventive maintenance policy that considered imperfect repair and postponed replacement. Their research recognized that not all repairs restore equipment to a "good-as-new" state, thus incorporating imperfect maintenance into decision models. By allowing postponed replacement, they balanced cost and system reliability, offering a more realistic approach to long-term equipment upkeep. The two-phase policy provided flexibility in managing both minor maintenance tasks and major replacement decisions, reflecting practical constraints in real-world operations. Dinh et al. (2020) demonstrated how structural dependencies significantly influence maintenance strategies, underscoring the need to incorporate system-level interactions in repair facility design and scheduling. Hao et al. (2020) showed that inspection reliability directly affects the efficiency of condition-based maintenance policies. This study underscored the practical issue of imperfect information in maintenance decisions, which parallels the challenge of imperfect spares in manufacturing systems. Liu et al. (2021) highlighted the complexity of considering multiple degradation factors simultaneously and showed how imperfect inspections amplify system vulnerability. Their contribution was crucial in extending single-dimensional reliability models into more realistic, multi-dimensional domains. Shekhar et al. (2021) provided a mathematical framework for evaluating machining reliability, showing that even small imperfections significantly reduce system performance. This aligned with the broader literature stressing that repair and spare imperfections cannot be ignored in performance analysis. Zhang et al. (2021) conducted a comprehensive literature review on spare parts inventory management. Their work synthesized previous findings, highlighting major challenges such as demand uncertainty, cost trade-offs, and the increasing complexity of global supply chains. They also categorized different modeling approaches, from stochastic optimization to machine learning applications, and identified research gaps, particularly in sustainability and circular economy perspectives. This review served as a foundation for future studies aiming to align spare parts inventory with emerging industry needs. Sun et al. (2022) investigated optimization of after-sales services by considering spare parts consumption alongside repairman travel. Their research addressed a practical service problem where customer satisfaction depends not only on spare parts availability but also on response time. By jointly optimizing logistics of spare parts and technician movements, they showed significant improvements in service efficiency and customer satisfaction. This study broadened the scope of spare parts research beyond production systems to include service operations, underscoring the importance of customer-centric decision making. Zhang et al. (2022) bridged dependence modeling with imperfect maintenance observations, emphasizing that realistic reliability models must capture both component interdependencies and inspection limitations. He and Gao (2023) focused on the joint optimization of preventive maintenance and spare parts ordering, incorporating imperfect fault detection into their model. They recognized that in many real systems, diagnostic tools do not always detect failures perfectly, leading to potential risks of undetected faults. Their model optimized both the timing of maintenance activities and the quantity of spare parts ordered under imperfect information. The findings demonstrated that accounting for detection imperfections yields more robust and cost-effective policies, bridging the gap between theoretical models and practical uncertainties in system maintenance.

Model Description

(i) A manufacturing system requires one operational machine.
(ii) One main machine is in use, with imperfect spare(s) available.
(iii) Failures occur according to an exponential distribution with rate **λ**.
(iv) Repairs occur with rate μ (repair facility).
(v) Imperfect spares may fail at a higher rate $\lambda_s > \lambda$.
(vi) States are defined by the number of operational machines and repair activity.

We denote:

$P_i(t)$: Probability that the system is in state iii at time t.

$\sum_i P_i(t) = 1$ (Normal condition). (1)

BALANCE EQUATIONS

If n spares and r repair facilities are included, the equations extend to:

For state i (where i machines are failed):

$\lambda_{i-1}P_{i-1} + \mu_{i+1}P_{i+1} - (\lambda_i + \mu_i)P_i, i = 0,1,2,\dots,n$ (2)

Where:

λ_i: total failure rate when iii components are failed.

μ_i: effective repair rate when iii components are under repair.

Let P_i be the steady-state probability of being in state i.

State 0 (all healthy except main in operation):

$-\lambda P_0 - 2\lambda_s P_0 + \mu P_1 + \mu P_3 = 0$ (3)

State 1 (main failed under repair, spare in use, one spare healthy):

$\lambda P_0 - (\mu + \lambda_s)P_1 + \mu P_5 = 0$ (4)

State 2 (main failed waiting, spare in use, one spare healthy):

$\lambda_s P_1 - \mu P_2 = 0$ (5)

State 3 (spare failed under repair, main in use, one spare healthy):

$\lambda_s P_0 - (\mu + \lambda)P_3 + \mu P_5 = 0$ (6)

State 4 (spare failed waiting, main in use, one spare healthy):

$\lambda P_3 - \mu P_4 = 0$

State 5 (main + spare failed, one under repair, other waiting, one in use):

$\lambda_s P_1 + \lambda P_3 - (\mu + \lambda_s)P_5 + \mu P_6 = 0$ (7)

State 6 (main under repair, both spares failed):

$\lambda_s P_5 - (\mu + \lambda_s)P_6 + \mu P_8 = 0$ (8)

State 7 (main waiting, both spares failed):

$\lambda_s P_6 - \mu P_7 = 0$ (9)

State 8 (all three failed, one under repair, two waiting):

$\lambda_s P_6 - (\mu + \lambda)P_8 + \mu P_9 = 0$ (10)

State 9 (all failed, all waiting worst case):

$\lambda P_8 - \mu P_9 = 0$ (11)

Normalization Condition

$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 = 1$ (12)

SOLUTION OF THE MODEL

We can represent these in the global balance equation form:

$\mathbf{P}Q = 0$ (13)

Where:

$P = [P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9]$

Q= infinitesimal generator matrix (transition rate matrix).

$$Q = \begin{bmatrix} -(\lambda + 2\lambda_s) & \mu & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & (\mu + \mu_s) & 0 & 0 & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & \lambda_s & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_s & 0 & 0 & -(\mu + \lambda) & 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_s & 0 & \lambda & 0 & -(\mu + \lambda_s) & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_s & -(\mu + \lambda_s) & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_s & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_s & 0 & -(\mu + \lambda) & \mu \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & -\mu \end{bmatrix}$$

Normalization Condition

$\sum_{i=0}^{9} P_i = 1$

(13)

Using Recursive Method, We obtain

$P_1 = \frac{\lambda+\mu k}{\mu+\mu_s} P_0, P_2 = \frac{\lambda_s(\lambda+\mu k)}{\mu(\mu+\mu_s)} P_0, P_3 = \frac{\lambda_s+\mu k}{\mu+\lambda} P_0, P_4 = \frac{\lambda_s(\lambda+\mu k)}{\mu(\mu+\lambda)} P_0, P_5 = kP_0, P_6 = \frac{\lambda_s}{\mu} kP_0$

$P_7 = \frac{{\lambda_s}^2}{\mu^2} kP_0, P_8 = \frac{{\lambda_s}^2}{\mu^2} kP_0, P_9 = \frac{\lambda {\lambda_s}^2}{\mu^3} kP_0$ (14)

Where $k = \frac{(\lambda+2\lambda_s) - \frac{\mu\lambda}{\mu+\lambda_s} - \frac{\mu\lambda_s}{\mu+\lambda_s}}{\mu^2\left(\frac{1}{\mu+\lambda_s} + \frac{1}{\mu+\lambda}\right)}$ (15)

Using normal condition

$P_0 \left[1 + \frac{\lambda+\mu k}{\mu+\mu_s} + \frac{\lambda_s(\lambda+\mu k)}{\mu(\mu+\mu_s)} + \frac{\lambda_s+\mu k}{\mu+\lambda} + \frac{\lambda_s(\lambda+\mu k)}{\mu(\mu+\lambda)} + k + \frac{\lambda_s}{\mu} k + \frac{{\lambda_s}^2}{\mu^2} k + \frac{{\lambda_s}^2}{\mu^2} k + \frac{\lambda {\lambda_s}^2}{\mu^3}\right] = 1$

$P_0 = \frac{1}{U}$

Where $U = 1 + \frac{\lambda+\mu k}{\mu+\mu_s} + \frac{\lambda_s(\lambda+\mu k)}{\mu(\mu+\mu_s)} + \frac{\lambda_s+\mu k}{\mu+\lambda} + \frac{\lambda_s(\lambda+\mu k)}{\mu(\mu+\lambda)} + k + \frac{\lambda_s}{\mu} k + \frac{{\lambda_s}^2}{\mu^2} k + \frac{{\lambda_s}^2}{\mu^2} k + \frac{\lambda {\lambda_s}^2}{\mu^3}$

PERFORMANCE MEASURES

(i) System Availability:

$A = 1 - (P_7 + P_8 + P_9)$ (13)

(ii) Mean Number under Repair: $E[R] = P_1 + P_3 + P_5 + P_6 + P_8$ (14)

(iii) Expected Downtime Probability:

$U = P_7 + P_8 + P_9$ (15)

NUMERICAL SIMULATIONS

Let $\lambda = 0.1, \lambda_s = 0.02, \mu = 0.1$

Table 1:Steady-State Probabilities

State	Description	Probability (P_i)
P_0	Main operating, spares healthy	0.5738
P_1	Main under repair, 1 spare in use	0.1123
P_2	Main waiting, 1 spare in use	0.0225
P_3	Spare under repair, main in use	0.1746
P_4	Spare waiting, main in use	0.0175
P_5	Main + 1 spare failed, 1 under repair	0.0773
P_6	Main under repair, both spares failed	0.0155

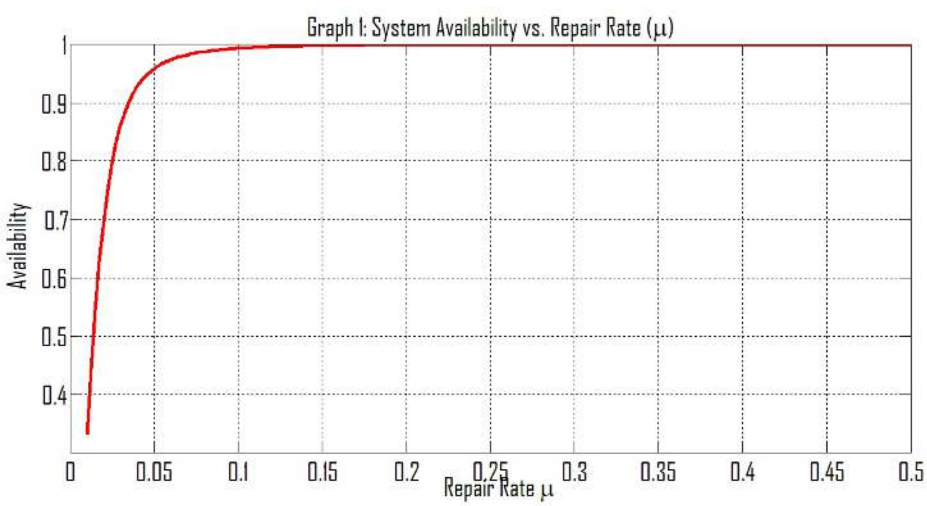
P_7	Main waiting, both spares failed	0.0031
P_8	All failed, 1 under repair	0.0031
P_9	All failed, all waiting	0.0003

System Availability=0.9935=99.35 %
The manufacturing system is operational almost all the time.
Mean Number under Repair=0.383
On average, about 0.38 machines are under repair at any given time.
Expected Downtime Probability=0.0065
The probability that the system is completely down is very small.

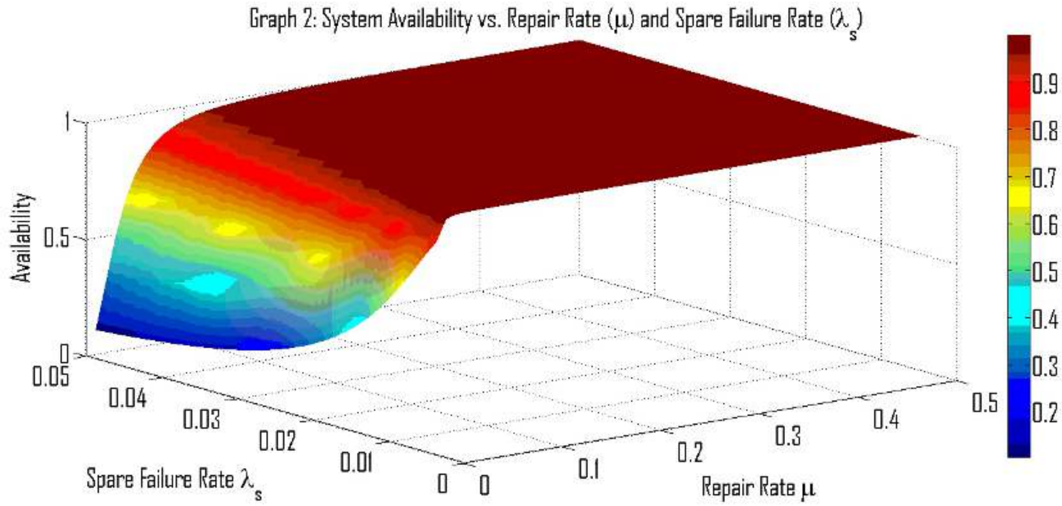
Table 2: Availability Summary

Spare Failure Rate (λ_s)	$\mu = 0.05$	$\mu = 0.1$	$\mu = 0.2$
$\lambda_s = 0.01$	0.9926898	0.999027	0.99988
$\lambda_s = 0.03$	0.8981887	0.9814626	0.99721
$\lambda_s = 0.05$	0.7416564	0.9373585	0.9891

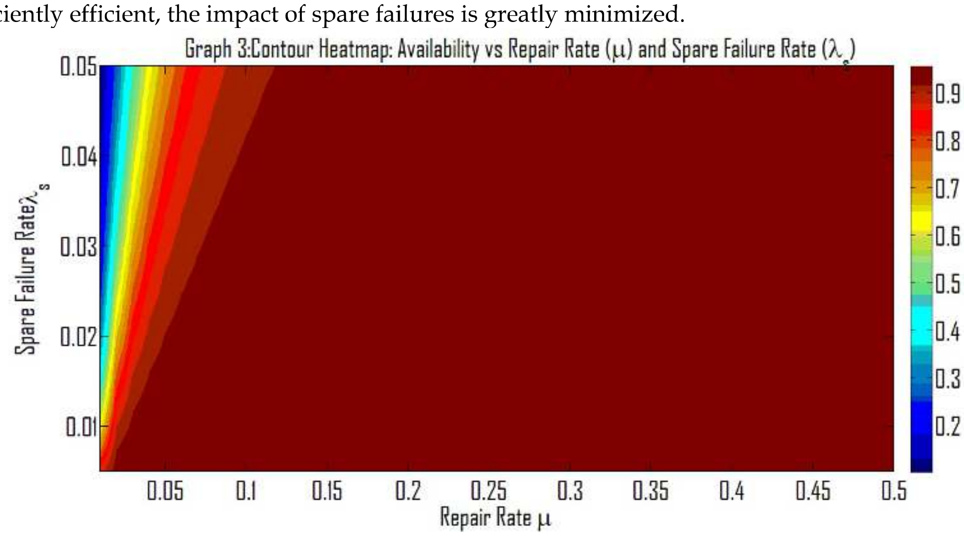
RESULTS AND DISCUSSION



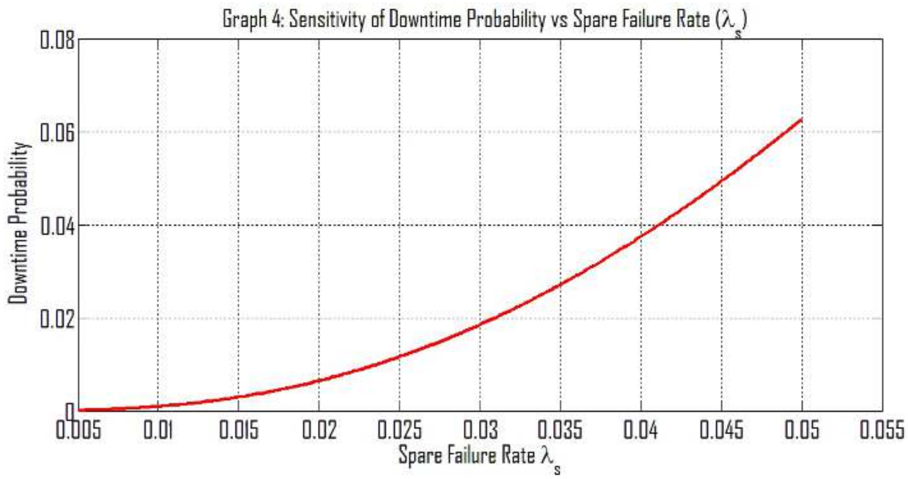
The graph (1) shows the variation of system availability with respect to the repair rate μ. It can be observed that availability increases rapidly as the repair rate increases from very small values, indicating that faster repairs significantly improve system reliability. After a certain threshold, the curve levels off and approaches a steady state near full availability, meaning that beyond this point, further increases in the repair rate have only a marginal effect. This behavior highlights the critical role of repair efficiency in maintaining high system performance, especially when repair rates are low, while also suggesting diminishing returns at higher repair rates.



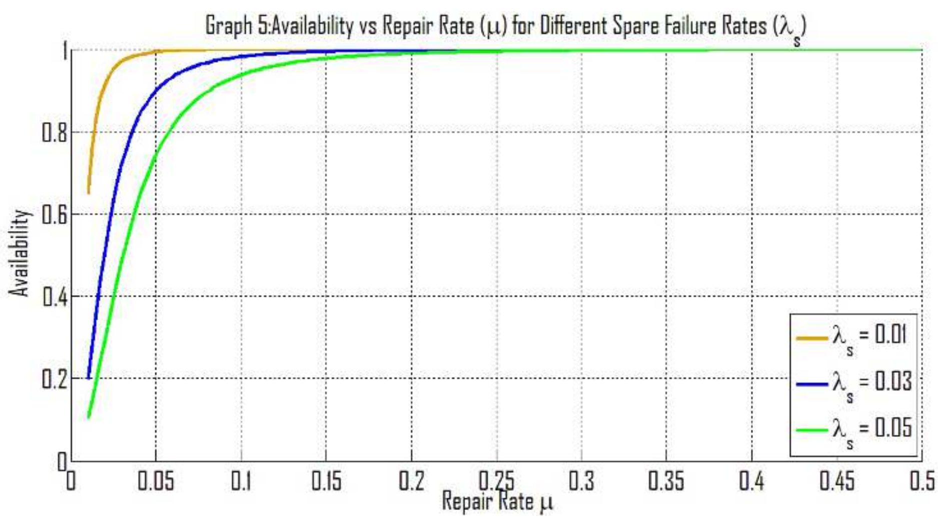
The graph (2) illustrates the combined effect of repair rate μ and spare failure rate λ_s on system availability. It is evident that availability is higher when the repair rate is large and the spare failure rate is small, indicating that reliable spares and efficient repair contribute significantly to system performance. At lower values of μ, availability drops sharply, especially when λ_s is high, showing that poor repair capability combined with unreliable spares leads to reduced system reliability. The surface gradually flattens near full availability for higher repair rates, suggesting that once the repair process becomes sufficiently efficient, the impact of spare failures is greatly minimized.



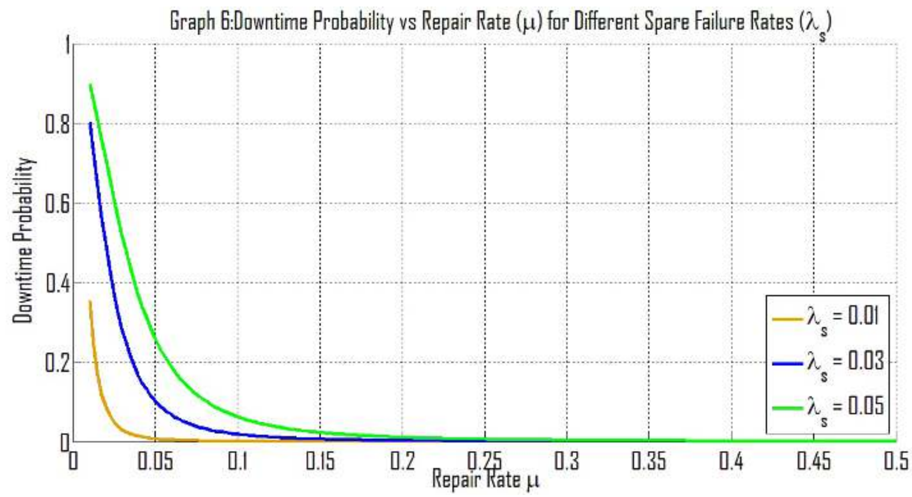
The contour heatmap (3) shows the relationship between system availability, repair rate μ, and spare failure rate λ_s. The red region represents high availability, while the blue and yellow shades indicate lower availability. It can be seen that availability remains close to one when the repair rate is sufficiently high, regardless of the spare failure rate. However, for lower repair rates, availability is highly sensitive to λ_s, with higher spare failure rates leading to reduced performance. This indicates that the combined effect of slow repairs and unreliable spares significantly lowers system reliability, whereas efficient repair can almost completely offset the impact of spare failures.



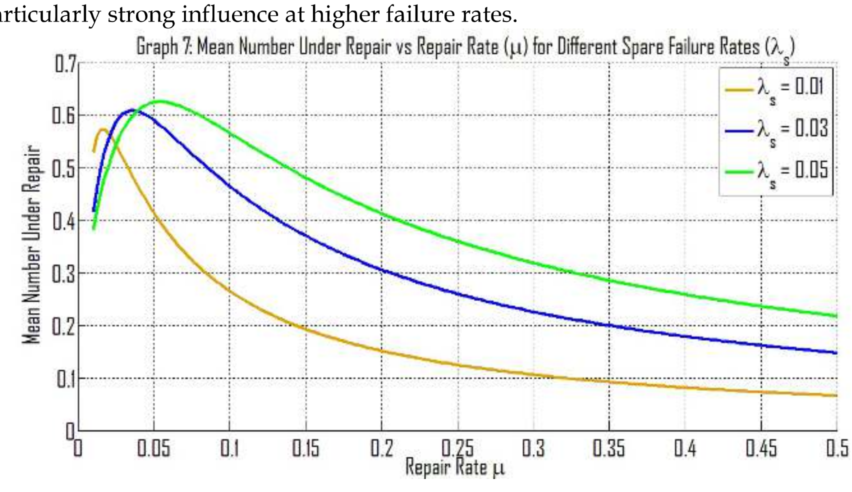
The graph (4) illustrates how downtime probability varies with spare failure rate λ_s. It shows a clear upward trend, where downtime probability increases steadily as the spare failure rate rises. At very low values of λ_s, the downtime probability is minimal, almost negligible, indicating that highly reliable spares contribute to maintaining system operation. As λ_s increases, the curve becomes steeper, highlighting that frequent failures of spares lead to a greater chance of the system being down. This demonstrates the strong sensitivity of system performance to the quality of spare units, where even moderate increases in λ_s can significantly reduce reliability.



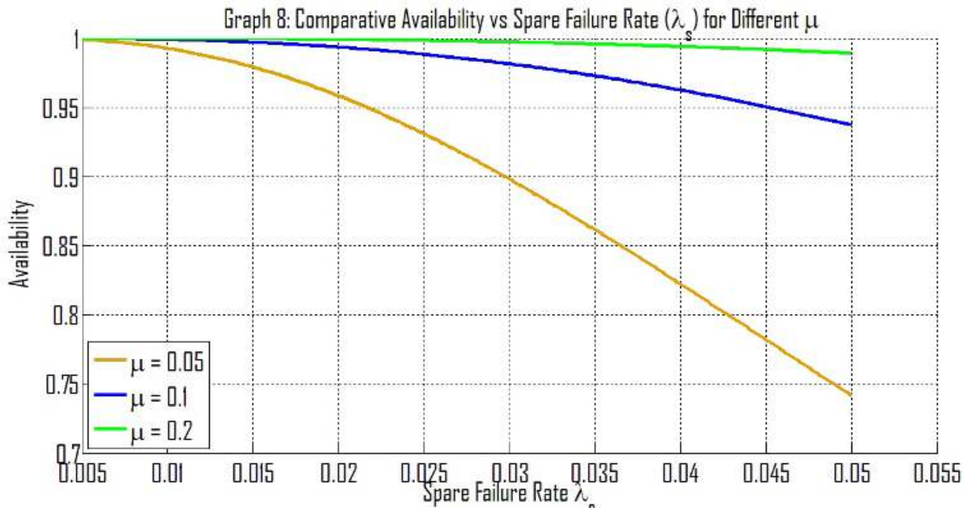
The graph (5) shows how availability changes with repair rate μ for three different spare failure rates λ_s . When λs is very low at 0.01, the system quickly reaches near perfect availability with only a moderate increase in repair rate. For higher λ_s values such as 0.03 and 0.05, availability starts lower at small μ, indicating that unreliable spares negatively affect performance. However, as μ increases, availability rises steadily for all cases and eventually approaches one, showing that efficient repair can largely compensate for the effect of spare failures. This highlights the balance between spare reliability and repair efficiency in ensuring high availability.



The graph (6) shows the relationship between downtime probability and repair rate μ for three different spare failure rates λ_s. It can be observed that when the repair rate is low, downtime probability is high, especially for larger λ_s values such as 0.05, where the curve starts near 0.9. As the repair rate increases, downtime probability drops sharply, demonstrating that efficient repair greatly reduces the likelihood of system unavailability. For low λ_s, such as 0.01, downtime probability is already relatively small and quickly approaches zero with even moderate repair rates. This indicates that both reliable spares and effective repair mechanisms are crucial for minimizing downtime, with repair efficiency having a particularly strong influence at higher failure rates.



The graph (7) illustrates the mean number of units under repair as a function of repair rate μ for three different spare failure rates λ_s. For all curves, the mean number under repair initially increases slightly at low μ, as failures accumulate faster than repairs. After reaching a peak, the values start to decline as μ increases, reflecting the system's ability to repair machines more quickly. The peak is higher and the curve remains elevated for larger λ_s values such as 0.05, indicating that unreliable spares create a heavier repair load. In contrast, when λ_s is small at 0.01, the repair load decreases rapidly with increasing μ and approaches very low values. This shows that both spare reliability and repair efficiency strongly influence the workload of the repair facility.



The graph (8) illustrates the relationship between system availability and the spare failure rate for three different repair rates (μ = 0.05, 0.1, and 0.2). As the spare failure rate increases, the overall system availability decreases for all repair rates, but the rate of decline varies depending on the value of μ. At a lower repair rate (μ = 0.05), the availability drops significantly with increasing spare failure rate, showing that the system is more vulnerable when repairs are slower. For moderate repair rates (μ = 0.1), the availability still decreases but at a slower rate, indicating better system resilience. When the repair rate is high (μ = 0.2), the availability remains close to one even at higher spare failure rates, highlighting that faster repair processes effectively counteract the negative impact of spare failures. This comparison emphasizes the importance of higher repair efficiency in maintaining system reliability under varying spare failure conditions.

8. Concluding Remarks: The analysis demonstrates that system performance in manufacturing environments is significantly influenced by both the quality of spare machines and the efficiency of repair facilities. Numerical results show that while imperfect spares increase the repair load and downtime risk, efficient repair processes can substantially mitigate these effects. High repair rates lead to near-perfect availability, even in the presence of unreliable spares, whereas slow repair mechanisms combined with high spare failure rates drastically reduce system reliability. Overall, the study highlights the importance of balancing spare reliability with repair capability to minimize downtime and ensure consistent operational performance in manufacturing systems.

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