CHAPTER-44

THE RESPONSE OF HEMATOCRIT ON BLOOD FLOW IN A STENOSED CYLINDRICAL ARTERY, AS PREDICTED BY THE WALBURN-SCHNECK MODEL

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Chapter ID: NSP/ICAAR-2023/A-44

ABSTRACT

Atherosclerosis, the buildup of fatty deposits and other substances in the arteries, causes the abnormal condition known as arterial stenosis. A heart attack may occur if the heart's blood supply is suddenly cut off. We have demonstrated in this study that a decrease in hematocrit causes a decrease in blood flow within the stenotic cylinder. We have also compared the pressure drop against the stenosis wall to the pressure drop against the unobstructed wall, and compared the maximum shear stress to the minimum shear stress at different stenosis thicknesses relative to the artery's radius to draw conclusions about the dynamics.

Keywords: *wall shear stress ratio, pressure drop, volumetric flow rate, hematocrit level, Walburn-*Schneck model

INTRODUCTION

The condition known as atherosclerosis is characterized by the accumulation of cholesterol-rich plaque in the arterial walls, which leads to stenosis (a narrowing of the artery) and a loss of flexibility in the arterial wall. Vascular illness is frequently the root cause of both heart attacks and strokes, which are ranked among the top three foremost causes of fatality across the globe. The typical cause of a stroke is the rupture or occlusion of a plaque that has formed inside of an artery in the brain. The lumen of the artery will become more constricted whenever there is both a variable wall shear stress (WSS) and turbulent flow.

Tashtoush and Magableh (2007) did research on the consequences of a magnetic field on blood flow in a multistenosed artery. They made the assumption that blood acted like a Newtonian fluid when conducting their research. Kenjeres (2008) released the results of a mathematical examination of blood flow in real arteries when the arteries were subjected to a sturdy nonuniform magnetic field. Researchers Habibi and Ghasemi (2011) looked into how the incidence of a magnetic field affected the amount of

P-ISBN: 978-93-88996-58-7 / E-ISBN: 978-93-88996-57-0

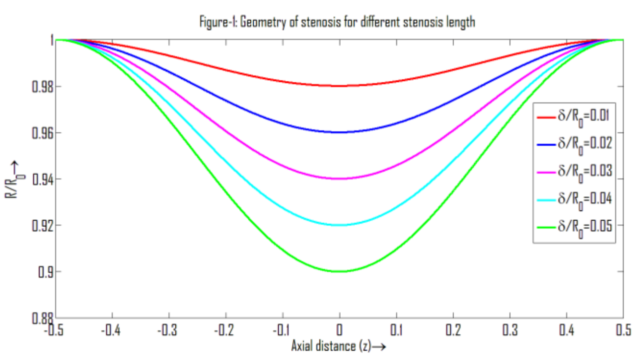
magnetic nanoparticles present in blood that did not contain a Newtonian fluid. In a study that was carried out numerically by Sankar et al. (2011), it was shown that the velocity of pulsatile laminar blood flow during a tiny symmetric stenosis decreased as the Hartmann number and amplitude increased. This was observed to be the case. Constant flow in mild and highly stenosed arteries was investigated by Alshare et al. (2013) using the Carreau viscosity model and a magnetic field. Moreno and Bhanagar (2013) built the model under settings that are representative of physiological flow, including fluctuations in flow (systole/diastole) and the evolution of flow from laminar to turbulent.

Bhatnagar et al. (2015) used a mathematical model that takes into account the scenario of velocity slip at the arterial wall in order to examine blood flow characteristics over an atherosclerotic arterial segment. Alshare and Tashtoush (2016) looked into the influence that the size of the artery and the strength of the magnetic field had on transient wall shear stress, mean shear stress, and pressure drop. Shah and Shukla (2017) did research on the quasi-conformal curvature tensor for Sasakian manifolds. In their research on the magnetized plasma sheath, Thakur et al. (2018) utilized a fluid hydrodynamic model that was positioned in a cylindrical coordinate system. Shah (2018) used the curvature criteria to Kenmotsu manifolds in order to investigate their behavior. Pokhrel et al. (2020) measured pressure, pressure drop on the wall, and shear stress on the wall in order to explore the dynamics of blood flow inside of a cylinder. Comparisons were made between the pressure drop against the wall with stenosis and the pressure drop beside the wall without stenosis. Additionally, comparisons were made between the maximum shear stress and the minimum shear stress for a variety of stenosis thicknesses and artery radii. Haider and Ahmad (2022) investigated the characteristics of blood flow that is not Newtonian in a stenosed elliptical artery. By treating the Rabinowitsch fluid as if it were blood, they were able to complete their work using the finite volume method. The non-Newtonian features of blood flow were studied by Shahzad et al. (2023) using the elliptical cross-section of a stenotic artery. The aforementioned mathematical model was made more linear by assuming a small stenosis, and the resulting equations were solved using the perturbation method.

MATHEMATICAL FORMULATION OF PROBLEM

As seen in Fig. 1, the radial structure of the surface of a cylinder can be described by the shape function $R(z)$, which is provided by the equation (1) below.

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + cos\frac{\pi z}{z_0}\right) \quad (1)$$



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Walburn-Schneck (1976) provides the following non-Newtonian model of blood based on hematocrit and shear rate.

$$\tau = p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\dot{\gamma}^{1-p_4 H} \quad (2)$$

where and $\dot{\gamma}$ and H are blood shear strain rate and hematocrit level, and p_1, p_2, p_3, p_4 are constants equal to 0.000797, 0.0608, 377.7515, and 0.00499, respectively, for a complete blood count.

The solution to equation (3) is

$$-\frac{dw}{dr} = \dot{\gamma} = \left[\frac{Pr}{2p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)}\right]^{1/1-p_4 H} \quad (3)$$

The limits are defined as

$$w = \begin{cases} 0 & wheen\ r = R(z)\ and - z_0 \le z \le z_0 \\ 0 & r = R_0\ and\ |z| \ge z_0 \end{cases} \quad (4)$$

The expression for blood velocity is obtained from equations (3) and (4) as

$$w = \left[\frac{P}{2p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)}\right]^{1/1-p_4 H} \frac{1-p_4 H}{2-p_4 H}\left[R^{\frac{2-p_4 H}{1-p_4 H}} - r^{\frac{2-p_4 H}{1-p_4 H}}\right] \quad (5)$$

The rate of flow in volume is calculated by

$$Q = \int_0^R 2\pi r w\, dr = \frac{\pi(1-p_4 H)}{4-3p_4 H}\left[\frac{P}{2p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)}\right]^{1/1-p_4 H} R_0^{\frac{4-3p_4 H}{1-p_4 H}}\left(\frac{R}{R_0}\right)^{\frac{4-3p_4 H}{1-p_4 H}} \quad (6)$$

The decreasing pressure that occurs over the duration of the stenosis is shown by

$$\Delta P = \int_{-Z_0}^{Z_0} P(z)dz = \int_{-Z_0}^{Z_0} 2p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4 H)}{\pi(1-p_4 H)}\right]^{1-p_4 H} \frac{1}{R^{4-3p_4 H}} dz$$

$$\Delta P = 2p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4 H)}{\pi(1-p_4 H)}\right]^{1-p_4 H} \frac{1}{R_0^{4-3p_4 H}} \int_{-z_0}^{z_0} \frac{1}{\left(\frac{R}{R_0}\right)^{4-3p_4 H}} dz \quad (7)$$

let us think about that

$$a = 1 - \frac{\delta}{2R_0}, b = \frac{\delta}{2R_0}, \phi = \frac{\pi z}{z_0} \Longrightarrow dz = \frac{1}{\pi} z_0 d\phi$$

So, the solution to (1) is

$$\frac{R}{R_0} = a - bcos\phi \quad (8)$$

Substituting Eq. (8) into Eq. (7), we get

$$\Delta P = 2p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4 H)}{\pi(1-p_4 H)}\right]^{1-p_4 H} \frac{1}{R_0^{4-3p_4 H}} \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{1}{(a-bcos\phi)^{4-3p_4 H}} \frac{1}{\pi} z_0 d\phi$$

$$\Delta P = 4p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4 H)}{\pi(1-p_4 H)}\right]^{1-p_4 H} \frac{1}{R_0^{4-3p_4 H}} \frac{z_0}{\pi} \int_0^{\pi} \frac{1}{(a-bcos\phi)^{4-3p_4 H}} d\phi \quad (9)$$

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When there is no stenosis present, the pressure decrease along the stenosis length is given by $\delta = 0$; but $f\left(\frac{\delta}{R_0}\right) = 1$.

$$(\Delta P)_p = 4p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4H)}{\pi(1-p_4H)}\right]^{1-p_4H} \frac{1}{{R_0}^{4-3p_4H}} \quad (11)$$

The ratio of pressure drops is defined as

$$\frac{\Delta P}{(\Delta P)_p} = \frac{z_0}{\pi}\int_0^{\pi} \frac{1}{(a-bcos\phi)^{4-3p_4H}} d\phi \quad (12)$$

Shear stress at the wall can be calculated as

$$\tau = \frac{1}{2}PR = p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4H)}{\pi(1-p_4H)}\right]^{1-p_4H} \frac{1}{R^{3-3p_4H}} \quad (13)$$

To the contrary, $f\left(\frac{\delta}{R_0}\right) = 1$ when $\delta = 0$ Shear stress along the length of the stenosis is

$$\tau_p = p_1 exp\left(p_2 H + \frac{p_3}{H^2}\right)\left[\frac{Q(4-3p_4H)}{\pi(1-p_4H)}\right]^{1-p_4H} \frac{1}{{R_0}^{3-3p_4H}} \quad (14)$$

The wall shear stress is proportional to

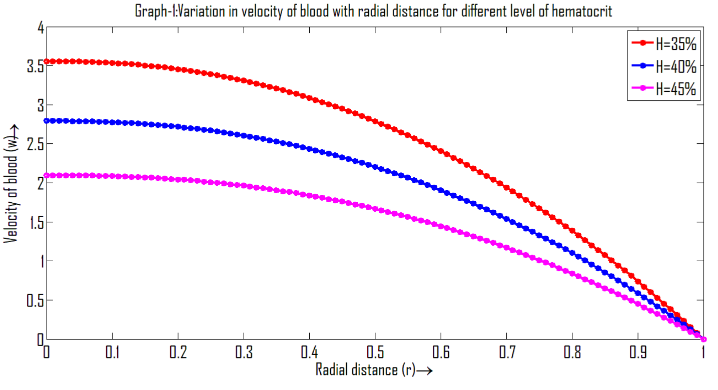
$$\tau_w = \frac{\tau}{\tau_p} = \frac{1}{\left(\frac{R}{R_0}\right)^{3-3p_4H}}$$

$$\tau_w = \frac{\tau}{\tau_p} = \frac{1}{\left(1-\frac{\delta}{2R_0}-\frac{\delta}{2R_0}cos\frac{\pi z}{z_0}\right)^{3-3p_4H}} \quad (15)$$

Maximum stress divided by minimum stress equals

$$\frac{\tau_{max}}{\tau_{min}} = \frac{1}{\left(1-\frac{\delta}{2R_0}-\frac{\delta}{2R_0}\right)^{3-3p_4H}} = \left(1-\frac{\delta}{R_0}\right)^{3p_4H-3} \quad (16)$$

RESULTS AND DISCUSSION



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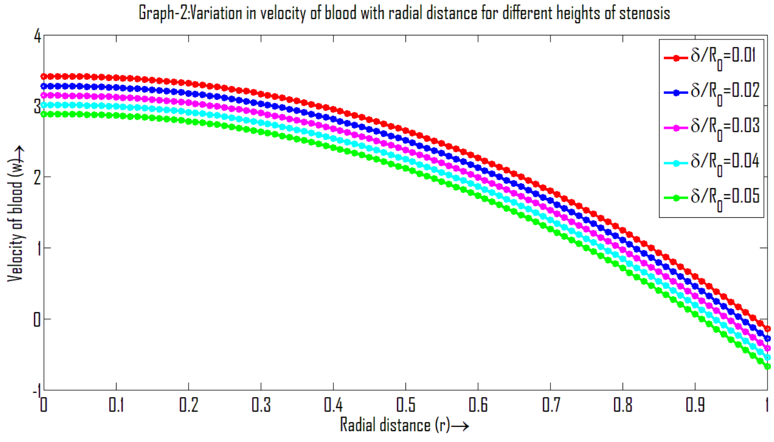

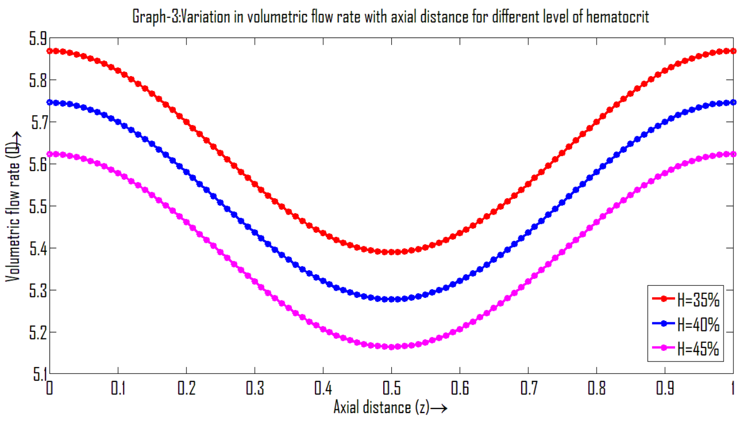

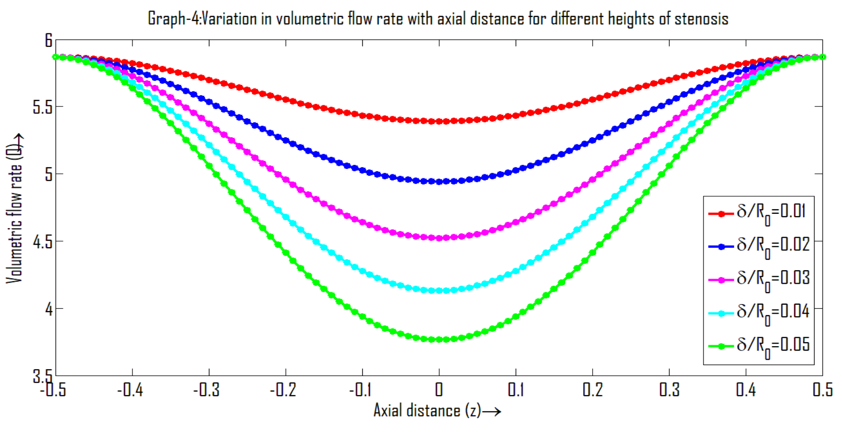

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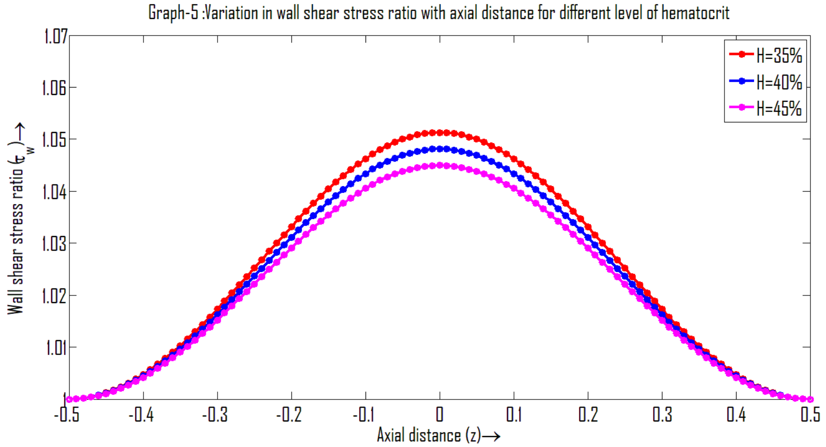

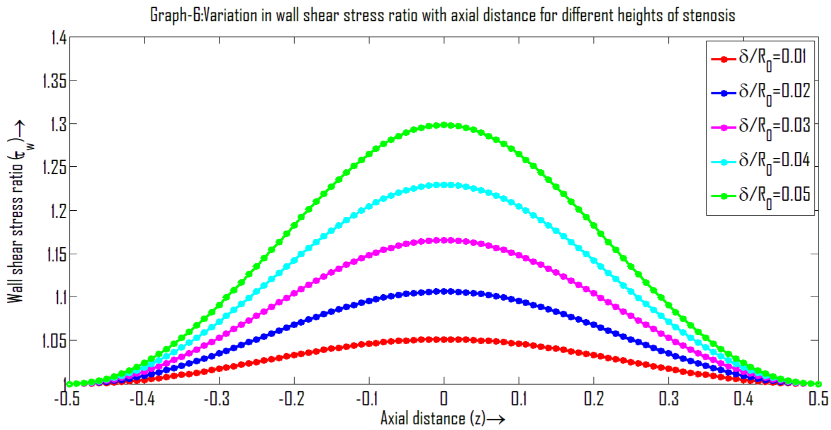

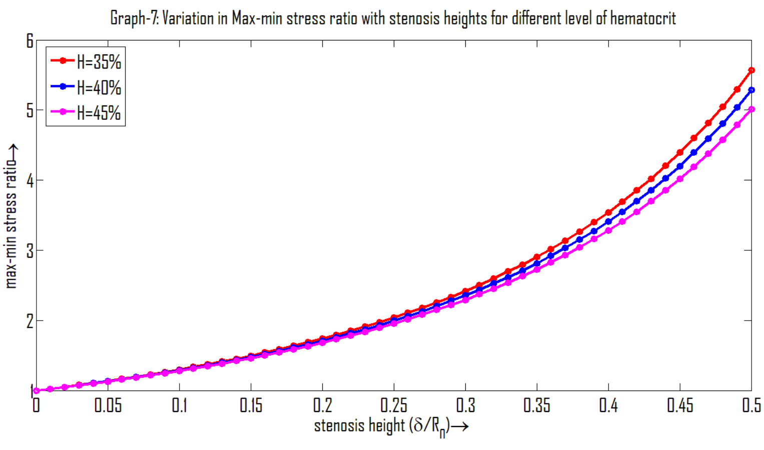

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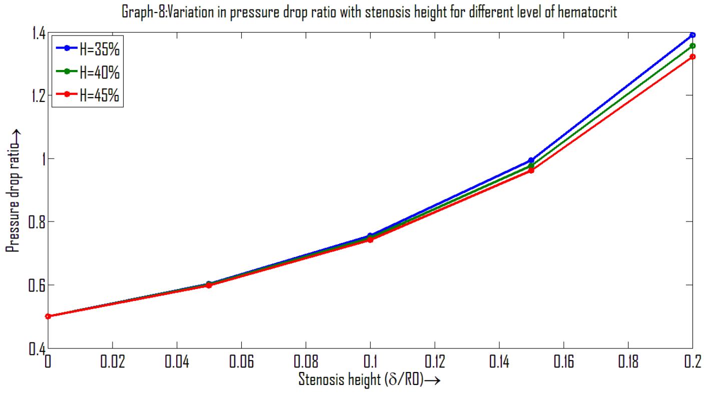

This research determined how stenosis height and hematocrit level affected blood flow through a cylindrical artery. Parameter values are taken into account along with their respective ranges as

$H = 35\% - 45\%$, $z_0 = 0.5$, $\frac{\delta}{R_0} = 0.01 - 0.05$ and $p_1 = 0.00797$, $p_2 = 0.0608$, $p_3 = 377.7515$, $p_4 = 0.00499$.

Equations (5) and (6) are used to calculate the blood velocity and volumetric flow rate, which are then displayed graphically in figures (1-4) for a range of hematocrit values and stenosis heights. As can be seen in graphs (1) and (2), blood velocity drops with increasing hematocrit and stenosis heights. The volumetric flow rate follows the same patterns as blood velocity, as can be seen in graphs (3) and (4).

When blood flows past an artery's endothelium, it creates a tangential force, known as wall shear stress. Blood velocity and viscosity, as well as the cube of the vessel's radius, all have a direct bearing on the magnitude of the shear force acting on its walls. Wall shear stress is highly sensitive to even modest variations in vessel radius.

Atherosclerotic plaque formation, development, and instability are all profoundly influenced by wall shear stress. Equations (13) and (15) illustrate how the wall shear stress of the permeable stenosed artery wall along the horizontal axis z changes as a function of stenosis height and hematocrit level. The outcomes have been plotted in graphs (5) and (6) against the z-axis. Wall shear stress in an artery is found to be significantly affected by both hematocrit and stenosis height. Wall shear stress is likewise found to be very height-dependent along the stenosis's growing axis, and is seen to rise as stenosis height increases. Wall shear stress is also observed to decrease with rising hematocrit. Graph (7) displays the relationship between the max-min stress ratio and stenosis heights and hematocrit. The maximum-minimum shear ratio correlates with wall shear stress. As illustrated in graph (7), the maximum shear stress relative to the minimum shear stress in the wall increases almost linearly with increasing stenosis thickness. In graph (8), we see how the hematocrit level affects the pressure drop throughout a range of stenosis heights. As hematocrit rises, the pressure drop experienced by the patient diminishes.

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CONCLUDING REMARKS

Here, we apply the Walburn-Schneck model to the investigation of blood flow dynamics in a stenotic artery in a cylindrical vessel. It was demonstrated that the hematocrit level and stenosis height influenced the velocity, volumetric flow rate, pressure drop, and wall shear stress ratio of the permeable wall. It has been shown that the velocity and volumetric flow rate of blood decrease with increasing hematocrit and stenosis height. Wall shear stress rartio is also reported to decrease with increasing hematocrit and decreasing stenosis height. Maximum to minimum stress ratio correlates with hematocrit and stenosis size in the same way as wall shear stress ratio does. These findings may be valuable to medical practitioners and bio mathematicians for the modeling of emerging issues and the prevention of cardiovascular disease.

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