# Chapter-38

# COSMOLOGICAL CONSTRAINTS ON DARK ENERGY: BIANCHI TYPE-I UNIVERSE AND BARROW HOLOGRAPHIC MODEL WITHIN SYMMETRIC TELEPARALLEL GRAVITY

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## *ABSTRACT*

*In this study, we undertake a detailed examination of the complex relationship between cosmological constraints on dark energy and the geometric characteristics of the universe, focusing specifically on a spatially homogeneous and anisotropic Bianchi Type-I space-time. Our investigation centers on the integration of Barrow holographic dark energy, a concept introduced by Barrow, wherein the infrared cut-off is aligned with Hubble's horizon within the framework of* $f(Q)$ *gravity, where the non-metricity* $Q$ *governs gravitational interaction and is expressed by the specific choice of* $f(Q)=\mu Q^2$ *(with* $\mu<0$ *as a constant). This theoretical model provides a promising avenue for scrutinizing the dynamics of dark energy in tandem with the influence of matter within the gravitational context. A key aspect of our analysis revolves around gravity's treatment within symmetric teleparallel gravity, where non-metricity mediates gravitational interaction. We adopt a specific constant parameter, denoted as, to characterize non-metricity. To elucidate the dynamics inherent in this framework, we undertake the challenge of deriving exact solutions to the field equations governing the universe's evolution. Central to our methodology is the exploration of the deceleration parameter, q, which is allowed to vary over time,* $t$ *, expressed as* $q=-\frac{\beta_1}{t^2}+\beta_2-1$*, where* $\beta_1$ *and* $\beta_2$ *are constants. We extend our investigation beyond mere solution determination, delving into the physical implications elucidated by key cosmological parameters. Notably, we analyze the equation of state parameter, the behavior of Barrow holographic dark energy, and the matter density. Furthermore, we scrutinize the skewness parameter, providing insights into the asymmetry present within cosmic structures. Through this comprehensive approach, we aim to deepen our understanding of the intricate interplay between dark energy dynamics, gravitational effects, and the geometric properties of the universe.*

***Keywords:*** *Cosmological Constraints, Bianchi Type-I Universe, Barrow Holographic Model, Symmetric Teleparallel Gravity, Homogeneous and Anisotropic Cosmology*

## INTRODUCTION

Throughout the course of astrophysics and cosmology, a significant amount of work has been made regarding the nature of dark energy and the implications it has for the universe.Dark energy, a mysterious form of energy, has been shown to play a substantial influence in determining the fate of the universe, particularly in terms of its rapid expansion. This has been determined through experiments. It is one of the most significant observations that is supporting the concept of dark energy that the acceleration of the expansion of the universe is being observed. Numerous astronomical researches, such as those involving type-Ia supernovae, large-scale structures, and studies of the anisotropy of the cosmic microwave background (CMB), have confirmed the prevalence of this phenomenon. When all of these observations are taken into consideration, it becomes clear that, contrary to what was anticipated, the expansion of the universe is actually gathering speed. At about 68.3% of the universe's total mass, dark energy is far and by the most important factor in the cosmic energy budget. About 26.8% is dark matter, another enigmatic component, and 4.9% is regular baryonic matter.

The distribution of dark energy over the universe demonstrates that it is a significant participant in the universe. Two metrics that have been created by scientists in order to gain a better understanding of the dynamics and evolution of the universe are the Hubble parameter $H$ and the deceleration parameter $(q)$. Both the Hubble parameter and the deceleration parameter shed light on the rate of expansion and slowing of the cosmos, respectively. Anisotropic cosmological models are one of several theoretical advancements prompted by the discovery and prevalence of dark energy. The standard cosmological theories and observations do not always agree with one another, and these models attempt to explain why this is the case, particularly with regard to the more primitive stages of the evolution of the cosmos.One possible explanation for cosmic acceleration is the existence of exotic, negatively-pressured matter forms that may overcome the effects of gravity. As a whole, dark energy research has added to our knowledge of the universe's make-up and dynamics while simultaneously challenging and inspiring both theoretical and observational cosmologists with fresh questions and insights. One of the most fascinating mysteries in contemporary astrophysics is the nature of dark energy, even though a lot of research has been made in this area.

Regarding the model of gravity inside the sight of dark matter and altered holographic Ricci Barrow energy (MHRDE) in locally rotationally symmetric (LRS) Bianchi type-I space-time, Shaikh et al. (2021) paid particular attention to the model. In order to achieve a truly appropriate arrangement of the field conditions, they considered volumetric power and exceptional extension regulations before making their decision. Within the framework of a scalar tensor hypothesis that was proposed by Saez and Ballester, Santhi and Sobhanbabu (2021) constructed connected and non-communicating Tsallis holographic Barrow energy models. These models were constructed in an anisotropic and homogeneous Bianchi type space time. Within the framework of $f(Q)$ gravity, where the non-metricity $Q$ is accountable for the gravitational interaction for the particular selection of $f(Q)=\lambda Q^2$, Koussour et al. (2022) addressed a spatially homogeneous and anisotropic Bianchi type-I space-time that is accompanied by Barrow holographic dark energy, where the infrared cut-off is the Hubble's horizon. In an anisotropic Bianchi type-I Universe, where the gravitational interaction is caused by the non-metricity scalar $Q$, Koussour et al. (2022) introduced Barrow holographic dark energy, where the infrared cut-off is the

Hubble horizon, as recently proposed by Barrow. This is all within the framework of $f(Q)$ symmetric teleparallel gravity. Using Barrow holographic dark energy, Sharma et al. (2022) found the state equation for the Barrow holographic energy density in a flat FLRW cosmological model. In a flat $f(R,T)$ Universe, they determined how the Barrow holographic dark energy corresponds to quintessence, k-essence, and dilation scalar field models. Srivastava et al. (2022) examined the holographicdark energy (BHDE) model involving IR cutoff as the Hubble skyline behind the scenes of a level Friedmann-Lemaitre-Robertson-Walker universe. The deceleration boundary shows the universe development from decelerated to sped up stage. Bahamonde et al (2023) gave an extensive prologue to how teleparallel calculation is created as a check hypothesis of interpretations along with the wide range of various properties of measure field hypothesis. In order to comprehend the universe's evolution, Bishi and Lepse (2023) built cosmological models of dark energy using Lyra's geometry for the Bianchi-I space time.

A model known as the non-level Kaniadakis holographic dark energy (KHDE) was created by Kumar and colleagues in the year 2023. Kaniadakis boundary and a boundary are elements that are utilized in the model. In order to illustrate the transformative manner in which the cosmos behaves, they were able to acquire the induction of the differential condition for the KHDE thickness boundary equation. Within the scope of this review, Ugale and Dhore (2023) conducted an analysis of the Bianchi type-I Barrow energy model in the context of the hypothesis of gravity. Through the utilization of constant deceleration border, they conducted an analysis of Bianchi type-I arrangements.

## EQUIVALENT TELEPARALLEL FIELD EQUATIONS IN METRIC AND SYMMETRIC SYSTEMS:

The source $S$ is provided by in $f(Q)$ as follows:

$$S = \int \left[ \frac{f(Q)}{2} + L_m \right] d^4x\sqrt{-g} \tag{1}$$

This is where the matter Lagrangian density $L_m$ is defined, $g$ is the determinant of the metric tensor $g_{\mu\nu}$ and $f(Q)$ is an arbitrary function of the non-metricity $Q$. The matter energy-momentum tensor $\left(T_{\mu\nu}\right)$ and the Barrow holographic dark energy $\left(\bar{T}_{\mu\nu}\right)$ are defined as.

$$T_{\mu\nu} = diag[-1,0,0,0]\rho_m \tag{2}$$

$$\bar{T}_{\mu\nu} = diag[-1,\omega_B,(\omega_B+\delta),(\omega_B+\delta)]\rho_B \tag{3}$$

where $\rho_m$ represents the energy density of matter, $\rho_B$ represents the energy density of Barrow holographic dark energy, and $p_B$ represents the pressure of Barrow holographic dark energy. The equation of state parameter is defined as $\omega_B = \frac{p_B}{\rho_B}$. The parameter δ in the equation represents the deviations from the equation of state parameter in two directions. We analyze the spatially uniform and anisotropic Bianchi type-I space-time using the provided metric.

$$ds^2 = -dt^2 + R^2dx^2 + S^2(dy^2+dz^2) \tag{4}$$

The metric potentials of the cosmos are represented by $R(t)$ and $S(t)$, respectively by these.

$$\frac{f}{2}+f'\left[4\frac{\dot{R}\dot{S}}{RS}+2\left(\frac{\dot{S}}{S}\right)^2\right]=\rho_m+\rho_B \tag{5}$$

$$\frac{f}{2}-f'\left[-2\frac{\dot{R}\dot{S}}{RS}-2\frac{\ddot{S}}{S}-2\left(\frac{\dot{S}}{S}\right)^2\right]+2\frac{\dot{S}}{S}\dot{b}f''=-\omega_B\rho_B \tag{6}$$

$$\frac{f}{2}-f''\left[-3\frac{\dot{R}\dot{S}}{RS}-\frac{\ddot{R}}{R}-\frac{\ddot{S}}{S}-2\left(\frac{\dot{S}}{S}\right)^2\right]+\left(\frac{\dot{R}}{R}+\frac{\dot{S}}{S}\right)\dot{b}f''=-\left(\omega_B+\delta\right)\rho_B \tag{7}$$

It is where the non-metricity scalar $Q$ for Bianchi type-I space-time is defined.

$$Q=-2\left(\frac{\dot{S}}{S}\right)^2-4\frac{\dot{R}\dot{S}}{RS} \tag{8}$$

## THE RESOLUTION OF FIELD EQUATIONS AND MODELS OF THE COSMOS:

Currently, we are only able to obtain three independent field equations, each of which has seven unknown parameters, namely $R, S, \rho_m, \rho_B, \omega_B, \delta, Q$ . Consequently, it is not possible to determine the system of equations (5)-(7); other equations relating these parameters are necessary for the solution of this system explicitly. Several different hypotheses are used by the various investigators in the published research in order to solve this system. Several competing hypotheses have been advanced by researchers in the literature in an effort to decipher this system: The model's shear scalar $\left(\sigma^2\right)$ is directly related to the scalar expansion $(\theta)$, resulting in a connection between metric potentials as

$$R=S^n \tag{9}$$

Where $n$ is a positive constant that is responsible for the anisotropic behavior of space-time, and where $n's$ value is not equal to one. By studying the quadratic form of the $f(Q)$ function, which is represented by the equation $f(Q)=\mu Q^2$ , where $\mu<0$ is a constant, the solution to the field equations can be successfully achieved.

There is a relationship between the scale factor $(a)$ and the spatial volume $(V)$ of the space-time (4).

$$V=a^3=RS^2 \tag{10}$$

The selection of the letter $q$ in the form

$$q=-\frac{\beta_1}{t^2}+\beta_2-1 \tag{11}$$

Presented in the form of the Hubble parameter

$$H = \frac{t}{\beta_2 t^2 + \beta_1} \tag{12}$$

The scale factor $a = a_0 \left(\frac{\beta_2}{\beta_1}\right)^{\frac{1}{2\beta_2}} \left(t^2 + \frac{\beta_1}{\beta_2}\right)^{\frac{1}{2\beta_2}}$ (13)

$$V = a^3 = RS^2$$

$$R = a_0^{\frac{3n}{n+2}} \left(\frac{\beta_2}{\beta_1}\right)^{\frac{3n}{2\beta_2(n+2)}} \left(t^2 + \frac{\beta_1}{\beta_2}\right)^{\frac{3n}{2\beta_2(n+2)}} \tag{14}$$

$$S = a_0^{\frac{3}{n+2}} \left(\frac{\beta_2}{\beta_1}\right)^{\frac{3}{2\beta_2(n+2)}} \left(t^2 + \frac{\beta_1}{\beta_2}\right)^{\frac{3}{2\beta_2(n+2)}} \tag{15}$$

$$Q = -2\left(\frac{\dot{S}}{S}\right)^2 - 4\frac{\dot{R}\dot{S}}{RS}$$

$$= -2\left\{\frac{3t}{(n+2)(\beta_2 t^2 + \beta_1)}\right\}^2 - 4\frac{3nt}{(n+2)(\beta_2 t^2 + \beta_1)} \times \frac{3t}{(n+2)(\beta_2 t^2 + \beta_1)} Q = -2\left(\frac{\dot{S}}{S}\right)^2 - 4\frac{\dot{R}\dot{S}}{RS}$$

$$= -\frac{18t^2}{(n+2)^2(\beta_2 t^2 + \beta_1)^2} - \frac{36nt^2}{(n+2)^2(\beta_2 t^2 + \beta_1)^2} = -\frac{18t^2(n+2)}{(n+2)^2(\beta_2 t^2 + \beta_1)^2} = -\frac{18t^2}{(n+2)(\beta_2 t^2 + \beta_1)^2}$$

$$= -\frac{18}{n+2} H^2 \tag{16}$$

$$f = \mu Q^2 = \frac{324\mu t^4}{(n+2)^2(\beta_2 t^2 + \beta_1)^4} \tag{17}$$

The density of the Barrow holographic dark energy is described by this formula

$$\rho_B = KL^{\Delta - 2} \tag{18}$$

When $L$ represents the length of the horizon.

As the infrared cut-off of the system, we investigate the Hubble's horizon, which is represented by the equation $(L = H^{-1})$, where $H$ is the Hubble's parameter of the model. Accordingly, the energy density of Barrow holographic dark energy has the form of the following in symmetric teleparallel gravity configurations:

$$\rho_B = KH^{2-\Delta} \tag{19}$$

Combining equations (5) and (19), we get the following:

$$\rho_m = \frac{162\mu H^4}{(n+2)^2} - \frac{648\mu H^4}{(n+2)^2} - KH^{2-\Delta} = -\frac{486\mu H^4}{(n+2)^2} - KH^{2-\Delta} \tag{20}$$

It may be shown from equations (6) and (19) that

$$\omega_B = \frac{\mu H^{\Delta+2}}{K}\left[\frac{36}{n+2}\left\{\left(18n - \left((n+2)\beta_2 - 3\right) + 36\right)\right\} - \frac{162}{(n+2)^2} - \frac{216}{(n+2)^2}\left(\beta_2 - \frac{\beta_1}{t^2}\right)\right] + \frac{216\mu\beta_1 H^{\Delta}}{K(n+2)^2(\beta_2 t^2 + \beta_1)^2} \tag{21}$$

By incorporating the values of $\rho_B$ and $\omega_B$ from equations (19) and (21) into equation (7), we are able to calculate the skewness parameter as follows:

$$\delta = \frac{\mu H^{\Delta+2}}{K}\left[\frac{264}{(n+2)^2} - \frac{216n}{(n+2)^2}\left(\beta_2 - \frac{\beta_1}{t^2}\right) - \frac{36}{n+2}\left\{\frac{9(3n+2)}{(n+2)^2} + \frac{3(n+1)}{(n+2)(\beta_2 t^2 + \beta_1)^2} + \frac{9(n^2+1)}{(n+2)^2} + 18n - \left((n+2)\beta_2 - 3\right) + 36\right\}\right]$$
$$+ \frac{108\mu\beta_1(n-1)H^{\Delta}}{K(n+2)^2(\beta_2 t^2 + \beta_1)^2}$$

The following equation shows the link between the scale factor a and the red-shift $z$ .

$$a = \frac{a_0}{1+z}$$

where $a_0$ represents the values that are currently being used for the scale factor.

One can use the following to derive the cosmic time expression in terms of redshift $z$ :

$$t = \sqrt{\frac{\beta_1}{\beta_2}\left[\left(\frac{1}{1+z}\right)^{2\beta_2} - 1\right]}$$

A definition of the state-finder pair $\{r, s\}$ is as follows:

$$r = \frac{\dddot{a}}{aH^3}, s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}$$

The state-finder parameters are as follows: The variables $r$ and $s$ offer valuable insights into the deviation from the ΛCDM model and the acceleration of the universe. Within this context, the term $r$ refers to the acceleration of the universe, whereas the letter $s$ denotes the divergence from the model of the cosmological constant.

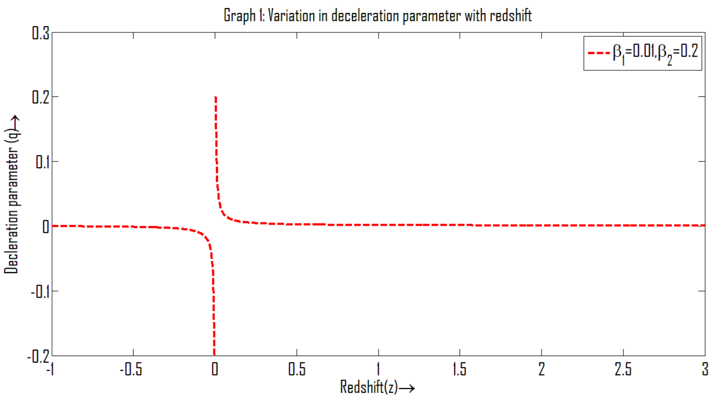
$$r = (2\beta_2 - 1)\left\{(\beta_2 - 1) - \frac{3\beta_1}{t^2}\right\}$$

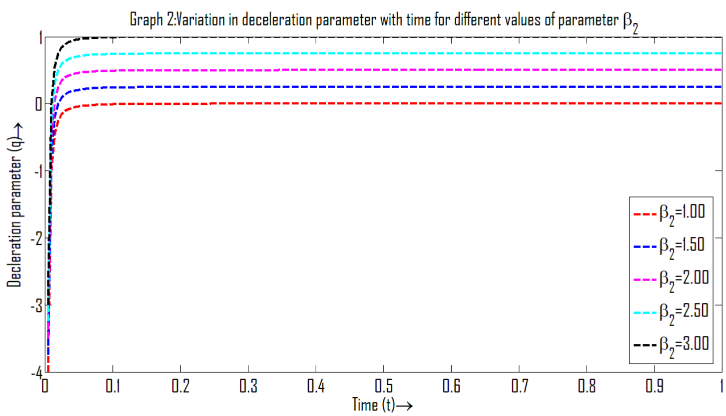
$$s=\frac{2}{3}\frac{(2\beta_2-1)\left\{(\beta_2-1)t^2-3\beta_1\right\}-t^2}{\left\{-2\beta_1+2\beta_2t^2-3t^2\right\}}$$

In the case where {r,s} is equal to {1,0}, we are able to obtain the model. Conversely, when {r,s} is equal to {1,1}, we encounter the cold dark matter limit. We also obtain a quintessence zone when the value of r is less than one, and a phantom region when the value of s is greater than zero.

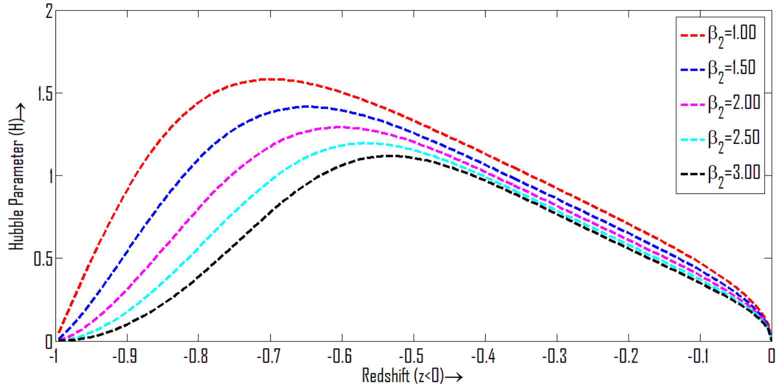
$$r\to-\infty, s\to 2\beta_2-1 \text{ as } t\to 0 \quad r\to(2\beta_2-1)(\beta_2-1), s\to\frac{2}{3}\frac{(2\beta_2-1)(\beta_2-1)-1}{(2\beta_2-3)} \text{ as } t\to\infty$$

## RESULTS AND DISCUSSION

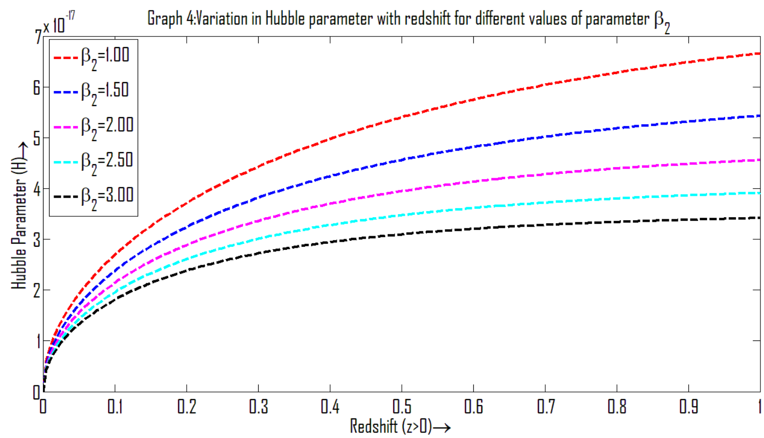


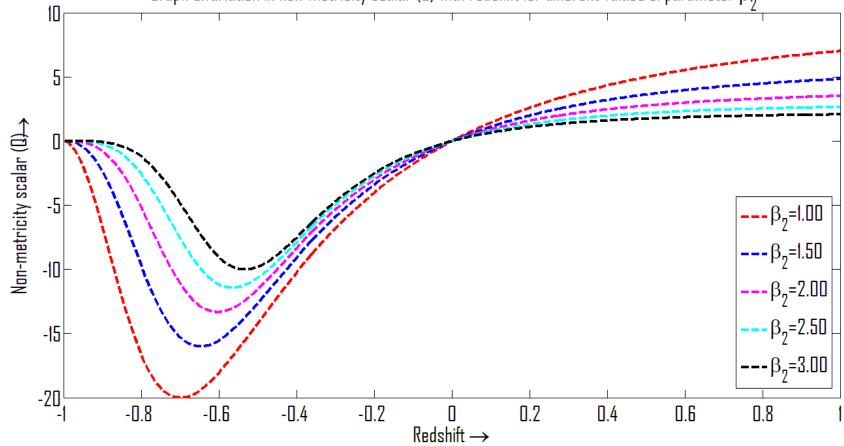
Graph 3:Variation in Hubble parameter with redshift for different values of parameter $\beta_2$

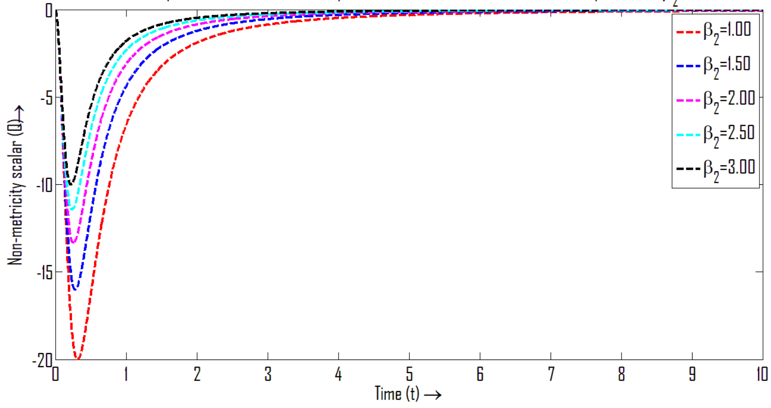



Graph 4:Variation in Hubble parameter with redshift for different values of parameter $\beta_2$

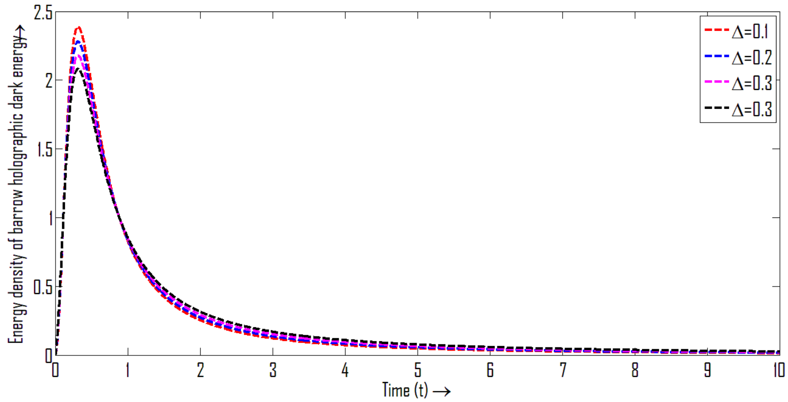



Graph 5:Variation in non-metricity scalar (Q) with redshift for different values of parameter $\beta_2$

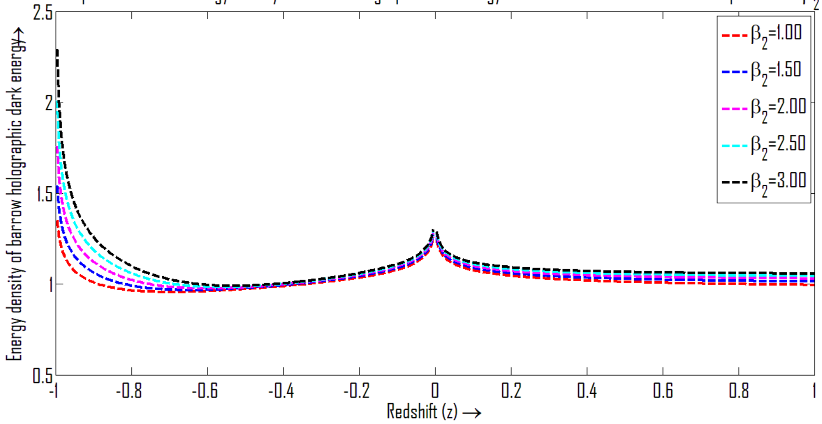




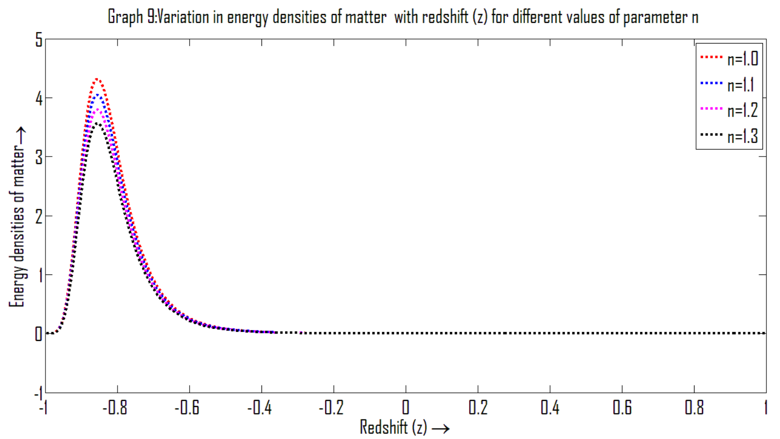

Graph 6:Variation in non-metricity scalar (Q) with time for different values of parameter $\beta_2$




Graph 7:Variation in energy density of barrow holographic dark energy with time for different values of barrow exponent




Graph 8:Variation in energy density of Barrow holographic dark energy with redshift for different values of parameter $\beta_2$




The graph (1) portrays connection among the deceleration parameter and the red-shift $(z)$ value. As the red shift grows, the $q$ begins with being valued positively to being valued negatively, and it converges towards 0 when z equals zero. This is something that can be shown using the graph (1). Consequently, We move from an early deceleration phase to a later acceleration phase $(q > 0)$ in our universe model to a contemporary acceleration phase $(q < 0)$ as the universe continues to accelerate. As a result of equation (11), the phase transition takes place when $\beta_2 = 1 + \frac{\beta_1}{t^2}$, which ultimately results in $q$ equal to zero. Consequently, our model aligns well with the latest observational evidence. For big enough values of $t$, the selection of q from equation (11) causes models to either expand or decelerate, as seen in graph (2).

This is caused by the fact that $q$ is equal to $\beta_2 - 1$ when t tends to infinity.Expanding and accelerating models for the case where $\beta_2$ equals 1.When expanding with continuous expansion and zero deceleration, the equation for $t^2$ is equal to $\frac{\beta_2 - 1}{\beta_1}$. Each of the graphs (3) and (4) displays a plot of the Hubble parameter versus the redshift (z) value. As the red shift grows, the graph (3) reveals that the has a positive value, and it converges towards when $z$ equals $-1$. This is something that can be witnessed. The graphs (4) demonstrate that the H has an increasing positive value with increasing red-shift, and it continues to converge in the direction of $0$ when $z$ equals zero. It has been demonstrated in graphs (5) and (6), respectively, how the behavior of non-metricity scalar acts as a relation to the redshift and time. When the parameters of the model are appropriately set, we plot the behavior of the Barrow holographic dark energy $(\rho_B)$ against time in graph (7). This graph displays the behavior of the dark energy. The density of Barrow holographic dark energy is observed to be an exact little amount that remains consistent, while the value of $\rho_B$ is observed in a way that is both positive and diminishing with respect to time. Plotting the behavior of the Barrow holographic dark energy $(\rho_B)$ and the energy density of matter ( $(\rho_m)$ with respect to the red-shift (z) for the appropriate values for the parameters of the model, correspondingly, , is shown in graphs (8) and (9). These graphs also include the Hubble's horizon cut-off. While the density of Barrow holographic dark energy is observed to attend a certain small constant value, the energy density of matter is observed to become null. Observations have shown that both $\rho_B$ and $\rho_m$ constitute positive functions that decrease with increasing red-shift.

**5. Concluding Remarks:**During the process of investigating the cosmological restrictions on dark energy within the framework of the Bianchi Type-I universe and the Barrow holographic model within given the circumstances of symmetric teleparallel gravity, a number of highly significant findings have been discovered. To begin, the employment of Bianchi Type-I cosmologies provides a valuable avenue for the investigation of the dynamics of dark energy in a universe that is anisotropic. Through the use of this methodology, it is possible to conduct a sophisticated investigation of the manner in which dark energy interacts with the geometry of the universe on both large and scales. In the second place, the incorporation of the Barrow Holographic Model within the context of symmetric teleparallel gravity offers a fresh theoretical framework for comprehending the characteristics of dark energy. This model offers a new viewpoint on the dynamics of dark energy and its implications for the entire cosmological development. It does this by taking into consideration gravitational theories and holographic concepts that go beyond General Relativity. The results of our investigation have led us to the conclusion that various cosmological frameworks offer perspectives that are complementary to one another about the behavior of dark energy. They give light on the complex relationship that exists between geometry, gravitational dynamics, and the unexplained features of dark energy. However, it is essential to keep in mind that our current comprehension of dark energy is still insufficient, and that additional study is required to deepen and expand upon these theoretical frameworks. For the purpose of limiting and validating these models, the availability of future observational data from studies such as large-scale surveys and investigations of the cosmic microwave background will be of critical importance. In conclusion, the investigation of cosmological restrictions on dark energy inside the Bianchi Type-I universe and the Barrow holographic model within symmetric teleparallel gravity constitutes a huge step forward in our effort to get to the bottom of the puzzles surrounding the evolution of the universe. Through the process of pushing the limits of theoretical frameworks and the constraints of observation, we are getting closer and closer to discovering the mysteries of dark energy and the part it plays in the formation of the universe.

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