CHAPTER: 29

IMPROVING THE EFFECTIVENESS OF INVENTORY CONTROL: A MODEL FOR DETERIORATING GOODS WITH MULTIPLICATIVE PRICE AND STOCK-DEPENDENT DEMAND FEATURES

VISHAL KUMAR PURWAR
Research Scholar, Department of Mathematics, St. John's College, Agra (India)
Affiliated to Dr. Bhimrao Ambedkar University, Agra (India)

Dr. SHARON MOSES
Associate Professor, Department of Mathematics, St. John's College, Agra (India)
Affiliated to Dr. Bhimrao Ambedkar University, Agra (India)

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ABSTRACT

For businesses that deal with perishable goods, efficient management of stock levels is key to minimizing losses and maximizing profits, making inventory management a crucial part of supply chain optimization. To tackle the challenges of managing products that are getting worse, this study suggests a new inventory model that incorporates stock-dependent demand, nonlinear holding costs, multiplicative pricing, and unique pricing techniques. According to this model, the demand for products is said to be affected by the amount of stock on hand and might decrease over time. In addition, because there are different costs associated with maintaining different amounts of inventory, the holding costs are thought to be nonlinear. The pricing technique is also multiplicative, so the price per unit changes depending on how many units are ordered. Maximizing earnings and easing the limitation of zero ending inventory while guaranteeing appropriate availability of items to meet customer demand is the goal of the model, which successfully balances order quantities, stock levels, and pricing methods. The suggested model is shown to be effective in controlling inventory of perishable commodities under different situations and parameter settings through numerical experiments. In making inventory decisions for perishable goods, the findings stress the significance of stock-dependent demand, nonlinear holding costs, and multiplicative pricing.

Keywords: *Inventory Management, Multiplicative Price, Stock-Dependent Demand, Nonlinear Holding Costs, Deteriorating Items*

1. INTRODUCTION

To maximize resources, decrease costs, and efficiently meet customer requests, good inventory control is crucial in the world of supply chain management. Dealing with things that are decaying, such perishable commodities or products with a limited shelf life, makes the process much more complex. Conventional inventory models frequently fail to appropriately handle the complications introduced by the ever-changing nature of these products. By include stock-dependent demand and nonlinear holding costs for deteriorating items, this research presents an integrated inventory management model that is specifically tailored to meet these difficulties. Due to their inherent tendency to depreciate with time, perishable goods offer a special set of difficulties for inventory management. When the pace of demand is affected by the existing level of stock, traditional models usually fail to account for this. If you want to optimize inventory levels to prevent stock outs or surplus inventory and precisely predict consumer behavior, you must incorporate this component. Furthermore, traditional models assume a linear relationship between holding costs and degrading products, whereas in practice, this relationship is typically non-linear. Factors like storage limitations, spoiling, or obsolescence can cause the price of holding large amounts of these commodities to escalate. Businesses may make better decisions about order amounts and stock levels when nonlinear holding costs are included in the model. This allows us to capture the true costs of inventory management for degrading commodities.

The rationale for maximizing profits rather than limiting expenses in an inventory system where the demand rate is dependent on the inventory level was explained by Chang (2004). In their study, Tayal et al. (2015) examined a production model in which the following parameters were taken into account: a constant deterioration rate of holding costs; an exponential demand rate; and the prohibition of shortages. For both products that are deteriorating and those that are not, Ghasemi (2015) created an economic production quantity model in which the holding cost is dependent on the ordering run length. With and without the ability to foresee shortages, as well as demand that is exponentially reliant on time and subject to variable degradation, Tripathi et al. (2017) developed an inventory model. Looking at the trade credit facility from the retailer's point of view, Panda et al. (2019) analyzed and solved all the potential model cases and subcases using the Lingua 10.0 program. The economic order quantity (EOQ) inventory model was discussed by Cárdenas-Barrón (2020) in the context of nonlinear stock dependent demand and nonlinear holding cost. They used a sensitivity analysis and numerical examples to show how the inventory models worked and what they found.

Ruidas et al. emphasized a theory for the production model with stock and price-dependent demand for perishable goods (2020). In this model, specific cost constraints are seen as range numbers and a profit function is found. Regarding green investment and price for non-instantaneously disintegrating commodities, Maihami et al. (2021) conducted a case study. They formulated the issue as a model for non-linear optimization and used analysis to determine the best course of action. In order to find the best time period for the inventory cycle and the lowest total average expenses, Shaikh and Gite (2022) made a stock model utilizing a fluffy strategy that requires some investment worth of cash into account. For stock-dependent demand that is price-sensitive and affected by greening efforts and degrading items, Shah et al. (2022) devised inventory policies that use all-units quantity discounts. Under full backlog, Singh et al.(2022) developed a production inventory model for perishable goods with a price-stock dependent demand. Zhang et al. (2022) investigated his approach for determining the order-up-to level, the reorder point, and the preservation technology investment for degrading items. This was done with the intention of maximizing the overall profit per unit time that the store makes. We modeled the issue mathematically, accounting for stock-dependent demand rate and holding cost. In their proposed integrated inventory model for environmentally friendly products, Singh and Ambedkar (2023) accounted for stock, selling price, and lifetime dependant demand, all while maintaining a steady deterioration rate in the face of inflation.

To handle the difficulties of managing perishable goods with demand that depends on stock levels and nonlinear holding costs, this research proposes an integrated inventory management model. Businesses can find the best inventory control policies that balance costs and service levels by using mathematical optimization approaches like simulation or dynamic programming.

2. NOTATIONS AND ASSUMPTIONS

In this research, a deterministic inventory model is introduced to analyze the degradation of products. The model incorporates the price and stock dependent demand rate, along with the nonlinear holding cost. The model is founded upon the subsequent assumptions and significations:

2.1. The lead time is negligible.

2.2. Replenishments occur immediately.

2.3. The fixed purchase cost, denoted as K, is constant and is known for each order.

2.4. The known and constant variables in this context are the selling price P and the variable buying cost (c) per unit. In essence, the analysis fails to account for both price reductions and the impact of inflation.

2.5. This deterioration rate θ $(0 < \theta < 1)$ is solely used for inventory that is currently in stock.

2.6. There is no variation in the replenishing cycles. Therefore, the planning horizon is defined as $[0, T]$, and only a normal planning cycle of length T is taken into account.

2.7. One way to express the demand function that is dependent on stock and price multiplied by both is

$$D(P, I) = a(1 - bP)(1 - cI) \text{`} \quad (1)$$

a stands for the highest possible demand, whereas b and c are constants that denote the price elasticity of demand and inventory elasticity of demand, respectively.

2.8. It is presumed that S, which is non-zero, is the starting and ending inventory. Time $t = 0$ is when the order quantity Q is added to inventory. The result is that $I(0) = Q + S$. Demand and degradation work together to deplete the inventory within the time interval $[0, T]$, $I(T) = S$, meaning that stock is at a minimum level of S at time. Figure 1 shows a graphical illustration of the inventory system.

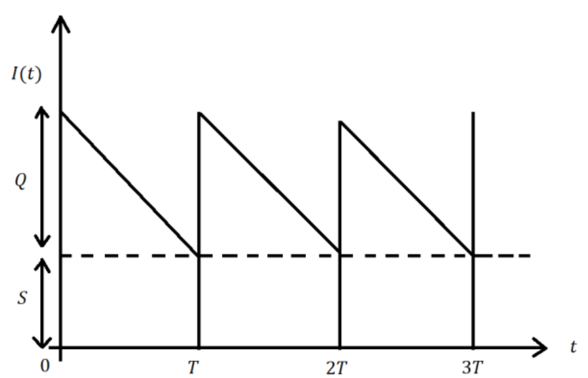


Figure 1: A visual representation illustrating the inventory system

3. MATHEMATICAL FORMULATION OF THE MODEL

To maximize the average net profit during the replenishment cycle, the mathematical task at hand is to calculate the ideal values of Q and S. This is the problem that needs to be solved.

When applied to the interval $[0,T]$, The differential equation governing the instantaneous states of I(t) is expressed as follows:

$$\frac{dI}{dt}+\theta I=-a(1-bP)(1-cI);0\leq t\leq T \tag{2}$$

$$\frac{dI}{dt}+(\theta+abcP-ac)I=abP-a;0\leq t\leq T \tag{3}$$

$$\frac{dI}{dt}+uI=v \tag{4}$$

Where $u=\theta+abcP-ac$ and $v=abP-a$

where $I(0)=Q+S$ is the beginning condition and $I(T)=S$ is the terminal condition. Equation (3) can be solved using the initial conditions provided, yielding

$$I\ =\frac{v}{u}+\left(Q+S-\frac{v}{u}\right)e^{-ut} \tag{5}$$

Through the expansion of the RHS of (4), the first-order approximation of u yields

$$I\ =\frac{v}{u}+\left(Q+S-\frac{v}{u}\right)(1-ut) \tag{6}$$

The result of plugging t into equation (5) is as

$$t=\frac{(Q+S-I)}{(Q+S)u-v}=\frac{Q+S}{(Q+S)u-v}\left[1-\frac{I}{Q+S}\right] \tag{7}$$

By finding $t's$ first derivative with regard to I, we obtain

$$dt=\frac{-1}{(Q+S)u-v}dI \tag{8}$$

The cycle time T is provided by since, as we can see from equation (6), $I(T)=S$.

$$T=\frac{Q}{(Q+S)u-v} \tag{9}$$

In the interval $[0,T]$, the total purchase cost (PC) is calculated as

$$PC=K+c_1Q \tag{10}$$

At the beginning of the period, it is important to take note of the fact that we are aware, based on Assumption (2.8), that the initial inventory is S units, and the order amounts are Q units. Since this is the case, the initial level of inventory for the period is a plus sign. The inventory level has returned to the S level at this moment. Demand and deterioration work together to bring the order quantities Q down to zero at time T. This is the result of the combined influence of the two factors. The total variable purchasing cost, shown by the symbol c_1Q, is calculated by adding the cost of deterioration for products that have already deteriorated to the cost of procurement for items that are currently in demand.

For the period $[0,T]$, the total revenue TR can be calculated as follows:

$$TR=P\int_0^T a(1-bP)(1-cI)dt \tag{11}$$

$$TR=\frac{aP(1-bP)}{(Q+S)u-v}\left[Q-cQS-\frac{cQ^2}{2}\right] \tag{12}$$

Following that, we will talk about two different kinds of costs associated with holding inventory.

3.1. A nonlinear cost of keeping that is dependent on time:

Assuming $n > 0$ and $h > 0$, we can write the cost of storing an inventory dI up to time t as $ht^n\, dI$ in this model. The entire cost of keeping inventory, denoted as HC, in the interval $[0, T]$, is therefore

$$HC = \int_S^{S+Q} h\left[\frac{Q+S}{(Q+S)u-v}\right]^n \left[1-\frac{I}{Q+S}\right]^n dI \tag{13}$$

$$HC = \frac{h(Q+S)^n}{[(Q+S)u-v]^n}\left[Q-\frac{n(Q+S)}{2}+\frac{n}{Q+S}\frac{S^2}{2}\right] \tag{14}$$

We may calculate the average net profit per unit of time as follows: $TP = TR - HC - PC$, where T is the time period in question.

$$AP = \frac{TP}{T} = \frac{TR-HC-PC}{T} \tag{15}$$

$$\left(\frac{\partial TR}{\partial Q}-\frac{\partial HC}{\partial Q}-\frac{\partial PC}{\partial Q}\right)T = (TR-HC-PC)\frac{dT}{dQ} \tag{16}$$

$$\left(\frac{\partial TR}{\partial S}-\frac{\partial HC}{\partial S}-\frac{\partial PC}{\partial S}\right)T = (TR-HC-PC)\frac{dT}{dS} \tag{17}$$

If we divide (15) by (16), we obtain

$$\left(\frac{\partial TR}{\partial Q}-\frac{\partial HC}{\partial Q}-\frac{\partial PC}{\partial Q}\right)\left(\frac{\partial T}{\partial S}\right) = \left(\frac{\partial TR}{\partial S}-\frac{\partial HC}{\partial S}-\frac{\partial PC}{\partial S}\right)\left(\frac{\partial T}{\partial Q}\right) \tag{18}$$

$$AP = \frac{TP}{T} = \frac{TR-HC-PC}{T} \tag{19}$$

$$\frac{\partial T}{\partial Q} = \frac{Su-v}{[(Q+S)u-v]^2} \tag{20}$$

$$\frac{\partial T}{\partial S} = -\frac{Qu}{[(Q+S)u-v]^2} \tag{21}$$

$$\frac{\partial PC}{\partial Q} = c_1 \tag{22}$$

$$\frac{\partial PC}{\partial S} = 0 \tag{23}$$

$$\frac{\partial TR}{\partial Q} = aP(1-bP)\left[\frac{1-cS-cQ}{(Q+S)u-v}-\frac{u\left(Q-cQS-\frac{cQ^2}{2}\right)}{[(Q+S)u-v]^2}\right] \tag{24}$$

$$\frac{\partial TR}{\partial S} = aP(1-bP)\left[\frac{Q\{2cv-(Qc+2)u\}}{[(Q+S)u-v]^2}\right] \tag{25}$$

$$\frac{\partial HC}{\partial Q} = h(B_1-B_2)$$

$$B_1 = \frac{\frac{S^2 n(n-1)(Q+S)^{n-2}}{2}+nQ(Q+S)^{n-1}+(Q+S)^n-\frac{n(n+1)(Q+S)^n}{2}}{[(Q+S)u-v]^n} \tag{26}$$

$$B_2 = nu[(Q+S)u-v]^{-n-1}\left[\frac{s^2 n(Q+S)^{n-1}}{2}+Q(Q+S)^n-\frac{n(Q+S)^{n+1}}{2}\right] \tag{27}$$

$$\frac{\partial HC}{\partial S} = h(D_1-B_2) \tag{28}$$

$$D_1 = \frac{\frac{s^2 n(n-1)(Q+S)^{n-2}}{2}+nQ(Q+S)^{n-1}+nS(Q+S)^{n-1}-\frac{n(n+1)(Q+S)^n}{2}}{[(Q+S)u-v]^n} \tag{29}$$

Equation (17) is solved by substituting the values of $\frac{\partial TR}{\partial Q}, \frac{\partial HC}{\partial Q}, \frac{\partial PC}{\partial Q}, \frac{dT}{dS}, \frac{\partial TR}{\partial S}, \frac{\partial HC}{\partial S}, \frac{\partial PC}{\partial S}$ and $\frac{dT}{dQ}$

$$\left[aP(1-bP)\left\{\frac{u\left(Q-cQS-\frac{cQ^2}{2}\right)}{[(Q+S)u-v]^2}-\frac{1-cS-cQ}{(Q+S)u-v}\right\}+h(B_1-B_2)+c_1\right]Qu=\left[aP(1-bP)\left\{\frac{Q\{2cv-(Qc+2)u\}}{[(Q+S)u-v]^2}\right\}-h(D_1-B_2)\right](Su-v) \quad (30)$$

Equation (15) may be used to find the values of $\frac{\partial TR}{\partial Q}, \frac{\partial HC}{\partial Q}, \frac{\partial PC}{\partial Q}, T, TR, HC, PC \text{ and } \frac{dT}{dQ}$.

$$\left[aP(1-bP)\left\{\frac{1-cS-cQ}{(Q+S)u-v}-\frac{u\left(Q-cQS-\frac{cQ^2}{2}\right)}{[(Q+S)u-v]^2}\right\}-h(B_1-B_2)-c_1\right]\frac{Q}{(Q+S)u-v}=$$

$$\left[\frac{aP(1-bP)}{(Q+S)u-v}\left\{Q-cQS-\frac{cQ^2}{2}\right\}-\frac{h(Q+S)^n}{[(Q+S)u-v]^n}\left\{Q-\frac{n(Q+S)}{2}+\frac{n}{Q+S}\frac{S^2}{2}\right\}-K+c_1Q\right]\frac{Su-v}{[(Q+S)u-v]^2}$$

$$\left[aP(1-bP)\left\{\frac{1-cS-cQ}{(Q+S)u-v}-\frac{u\left(Q-cQS-\frac{cQ^2}{2}\right)}{[(Q+S)u-v]^2}\right\}-h(B_1-B_2)-c_1\right]Q=\left[\frac{aP(1-bP)}{(Q+S)u-v}\left\{Q-cQS-\frac{cQ^2}{2}\right\}-\frac{h(Q+S)^n}{[(Q+S)u-v]^n}\left\{Q-\frac{n(Q+S)}{2}+\frac{n}{Q+S}\frac{S^2}{2}\right\}-K+c_1Q\right]\frac{Su-v}{(Q+S)u-v} \quad (31)$$

3.2. Holding costs that are nonlinear and depending on stock:

The definition of the holding cost per cycle in this scenario is as follows:

$$HC=\int_0^T hI^n\left\{\frac{-1}{(Q+S)u-v}\right\}dI=-\frac{h}{(Q+S)u-v}\left[\frac{I^{n+1}}{n+1}\right]_0^T=\frac{h\{(Q+S)^{n+1}-S^{n+1}\}}{[(Q+S)u-v](n+1)} \quad (32)$$

$$\frac{dHC}{dQ}=\frac{h(Q+S)^n}{u(Q+S)-v}-\frac{hu\{(Q+S)^{n+1}-S^{n+1}\}}{(n+1)[u(Q+S)-v]^2} \quad (33)$$

$$\frac{dHC}{dS}=\frac{h\{(Q+S)^n-S^n\}}{u(Q+S)-v}-\frac{hu\{(Q+S)^{n+1}-S^{n+1}\}}{(n+1)[u(Q+S)-v]^2} \quad (34)$$

$$\left[aP(1-bP)\left\{\frac{u\left(Q-cQS-\frac{cQ^2}{2}\right)}{[(Q+S)u-v]^2}-\frac{1-cS-cQ}{(Q+S)u-v}\right\}+\frac{h(Q+S)^n}{u(Q+S)-v}-\frac{hu\{(Q+S)^{n+1}-S^{n+1}\}}{(n+1)[u(Q+S)-v]^2}+c_1\right]Qu=\left[aP(1-bP)\left\{\frac{Q\{2cv-(Qc+2)u\}}{[(Q+S)u-v]^2}\right\}-\frac{h\{(Q+S)^n-S^n\}}{u(Q+S)-v}-\frac{hu\{(Q+S)^{n+1}-S^{n+1}\}}{(n+1)[u(Q+S)-v]^2}\right](Su-v) \quad (35)$$

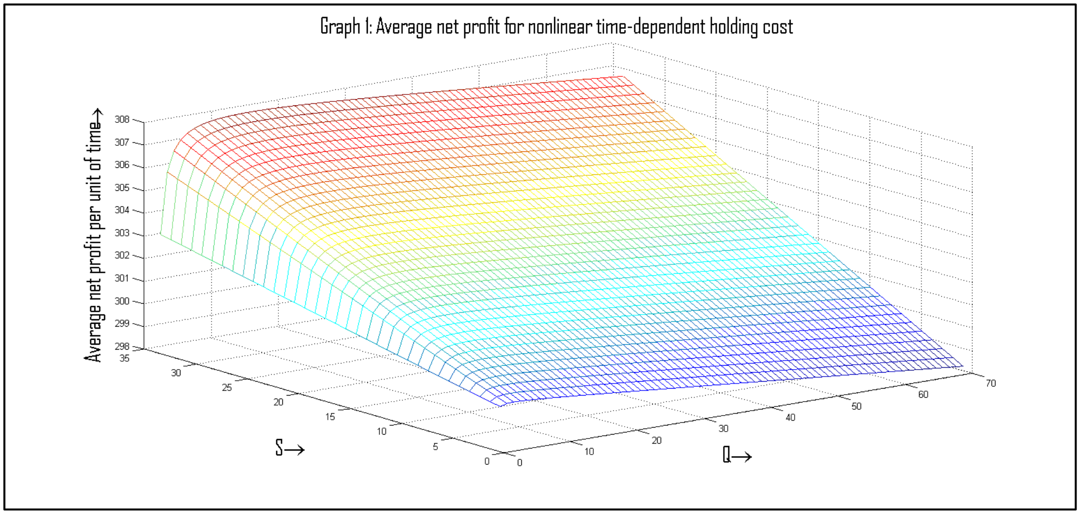
$$\left[aP(1-bP)\left\{\frac{1-cS-cQ}{(Q+S)u-v}-\frac{u\left(Q-cQS-\frac{cQ^2}{2}\right)}{[(Q+S)u-v]^2}\right\}-\frac{h(Q+S)^n}{u(Q+S)-v}+\frac{hu\{(Q+S)^{n+1}-S^{n+1}\}}{(n+1)[u(Q+S)-v]^2}-c_1\right]Q=\left[\frac{aP(1-bP)}{(Q+S)u-v}\left\{Q-cQS-\frac{cQ^2}{2}\right\}-\frac{h\{(Q+S)^n-S^n\}}{u(Q+S)-v}+\frac{hu\{(Q+S)^{n+1}-S^{n+1}\}}{(n+1)[u(Q+S)-v]^2}-K+c_1Q\right]\frac{Su-v}{(Q+S)u-v} \quad (36)$$

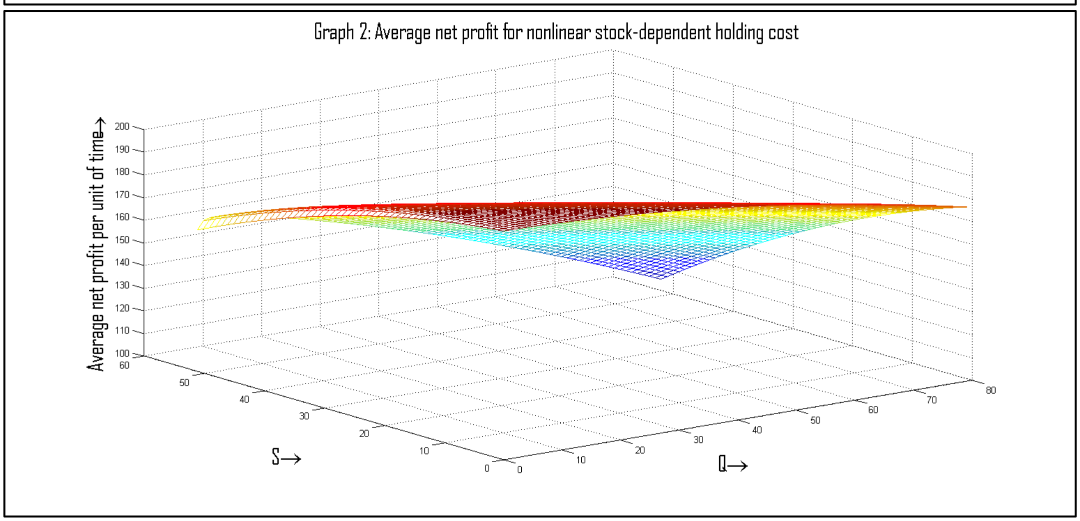
4. NUMERICAL EXPERIMENTS

For example, Let $a=0.01, b=0.03, c=2, h=0.01, K=5, \theta=0.05, P=3, n=2, c_1=0.02$. Through the utilization of the MATLAB 7.0 software, we are able to solve the equations (30) and (31) because they are both nonlinear and exceedingly challenging to solve. The computational result reveals the following optimal values for the nonlinear time-dependent holding cost, which are located in the intervals $Q\in[0,70]$ and $S\in[0,35]$.

S^* is equal to 32.5, Q^* is equal to 14.5, and AP^* is equal to 307.4666.

The nonlinear and exceedingly challenging equations (35) and (36) are solved using the MATLAB 7.0 program. For the nonlinear stock-dependent holding cost, the computational result reveals the following optimal values $Q \in [0,80]$ and $S \in [0,60]$, $S^* = 1, Q^* = 1, AP^* = 199.5571$





5. CONCLUDING REMARKS

Finally, while dealing with perishable goods, stock-dependent demand, and nonlinear holding costs, the integrated inventory management model offered in this research is a huge step forward. Taking into account the ever-changing demands of customers and the growing expenses of keeping inventory over time, this research has delved into the complexities of inventory control for products with a short shelf life or that are perishable. We have shown that it is important to account for demand changes based on the present stock level by including stock-dependent demand into the model. With this function, companies may optimize their inventory levels better, reducing the likelihood of stockouts and surplus while efficiently meeting client demand. Additionally, businesses are able to make better decisions about order amounts and stock levels when nonlinear holding costs are included, as they provide a more realistic depiction of the true costs of inventory management for degrading commodities. By utilizing mathematical optimization techniques like simulation or dynamic programming, we have demonstrated how companies can find the best inventory control rules that save costs while maintaining good service levels. Businesses may stay competitive and profitable by using these tactics to adapt to changing market conditions and make proactive decisions. Even though the integrated inventory management model provides helpful information and recommendations for companies, it's vital to note that there can be gaps in our understanding and opportunities for additional study. To make it more practical and effective in real-world scenarios, researchers should look into how the model works in other industries or add more variables like lead time variability or demand forecasting uncertainty.

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