CHAPTER-65

A COOPERATIVE GAME THEORY FRAMEWORK FOR COST ALLOCATION IN COLLABORATIVE LOGISTICS

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ABSTRACT

This paper develops a cooperative game theory framework for cost allocation in collaborative logistics, where multiple carriers or shippers share vehicles, depots and routes to reduce total transportation cost. Building on a vehicle routing–type optimisation model, the study first computes standalone and coalition costs for different groups of carriers and uses these to construct a characteristic function that captures the savings generated by any coalition. On this basis, several cost allocation rules are analysed, including the Shapley value, the nucleolus and a simple proportional rule. A numerical case study illustrates how collaboration restructures routes, lowers system-wide cost and redistributes individual costs among partners. The results highlight clear trade-offs between fairness and stability: Shapley and nucleolus allocations satisfy core and individual rationality conditions, whereas proportional sharing is easy to implement but may be unstable. The framework is complemented with a stability analysis and managerial discussion on sensitivity to input parameters, contractual design and scalability to larger coalitions, providing a transparent and analytically grounded tool to support the design of collaborative logistics agreements.

Keywords: *Collaborative logistics; horizontal collaboration; cooperative game theory; cost allocation; Shapley value; nucleolus; vehicle routing; core stability.*

1. INTRODUCTION

Collaborative logistics has emerged as a promising strategy for firms seeking to reduce transportation costs, improve asset utilization, and lower environmental impact by sharing vehicles, depots, and routes. However, once a joint plan is designed and total savings are realized, managers face a crucial and often contentious question: how should the common cost or benefit be divided among participating companies so that all perceive the outcome as fair and are willing to remain in the coalition? A cooperative game theory framework provides a rigorous way to address this challenge by modeling each firm as a player, quantifying the value generated by every possible coalition, and then deriving allocation rules that reflect contributions and bargaining power. Through solution concepts such as the Shapley value, the nucleolus, and various core-based methods, the framework links operational optimization models of routing and consolidation with economically sound cost-sharing outcomes. In this way, cooperative game theory not only clarifies the incentives behind collaboration but also offers managers transparent, analytically justified allocation schemes that can be embedded in contracts and long-term partnership agreements in collaborative logistics.

Maschler et al. (1979) provide the theoretical foundations for several cooperative game solution concepts, including the kernel and the nucleolus, and their geometric properties, which are central to any cost allocation framework that seeks both fairness and stability. Their work clarifies how the nucleolus lexicographically minimizes coalition dissatisfaction, offering a strong stability benchmark that is particularly relevant when designing allocation rules for collaborative logistics coalitions that must remain attractive to all subsets of partners over time. Building on this theoretical base, Frisk et al. (2010) demonstrate how cooperative game theory can be applied in practice to cost allocation in collaborative forest transportation, showing that sharing routes and vehicles can lead to significant savings, but also that the choice of allocation method (for example, Shapley value, nucleolus, or simpler rules) strongly affects perceived fairness and the willingness of firms to continue collaborating. Lozano et al. (2013) extend this line of work by explicitly adopting a cooperative game theory approach to allocate the benefits of horizontal cooperation, highlighting that properly defined characteristic functions and solution concepts can produce allocations that are both economically sound and compatible with the operational realities of shared logistics networks. Guajardo and Jörnsten (2015) focus on methodological rigor by identifying common mistakes in computing the nucleolus, warning that mis-specification of coalition values, incorrect handling of constraints, or inaccurate algorithmic implementations can lead to misleading results and potentially unstable agreements. Their insights are crucial for any applied framework that relies on advanced solution concepts, since incorrect computation would undermine both the theoretical and managerial credibility of cost allocations. Jouida et al. (2017) shift attention toward coalition-formation problems in sourcing contract design within supply networks, emphasizing that the decision to join or leave a coalition depends on how costs and benefits are shared and that game-theoretic models must be linked with realistic contract structures. This perspective supports the idea that, in collaborative logistics, cost allocation rules should not be considered in isolation but embedded in broader coalition formation and maintenance mechanisms. Wang et al. (2018) investigate two-echelon logistics delivery and pickup networks under integrated cooperation and fleet sharing, showing that operational integration across echelons can significantly reduce total system costs. Their work, however, stops short of providing a fully fledged cooperative game framework for distributing these savings among participants, thereby reinforcing the need for a systematic link between optimization models and game-theoretic allocation rules. Habibi et al. (2018) address collaborative hub location under cost uncertainty, demonstrating that cooperation in hub design can mitigate risk and improve performance, but again highlighting that sustainable collaboration requires robust and acceptable cost sharing among partners when costs are not deterministic. Together, these studies stress that realistic logistics settings often involve multi-echelon flows and uncertainty, which any comprehensive cooperative game framework should be able to accommodate through appropriate characteristic function design and sensitivity analysis. Defryn et al. (2019) concentrate on integrating partner objectives directly into horizontal logistics optimisation models, moving beyond purely cost-minimization views to incorporate equity and individual performance measures. Their results suggest that optimization models which explicitly consider partner-level objectives may reduce the gap between operational solutions and acceptable allocations, pointing to the value of combining cooperative game theory with multi-objective optimization. Pan et al. (2019) offer an extensive survey of horizontal collaborative transport, cataloguing solution methods, practical implementations, and barriers such as trust, data sharing, and allocation disputes. Their review underscores that one of the main obstacles to implementing collaborative transport schemes in practice is the absence of transparent and widely accepted cost-sharing mechanisms, underlining the practical relevance of a structured cooperative game theory framework. Badraoui et al. (2020) provide an exploratory empirical study of horizontal logistics collaboration in Morocco's agri-food supply chains, showing that although collaboration is perceived as beneficial, firms remain cautious due to concerns about fairness, trust, and the distribution of gains. This empirical evidence supports the argument that any proposed allocation framework must be both theoretically robust and easy for practitioners to understand and negotiate. Wang et al. (2020) study collaborative multi-depot logistics network design with time-window assignment, highlighting the additional coordination complexity that arises when timing constraints are combined with shared infrastructure and joint routing. Their work demonstrates the magnitude of potential cost savings from multi-depot collaboration but again leaves open the question of how to distribute network-level benefits among different depots and carriers, providing a direct motivation for using cooperative game theory to bridge from system-level optimisation to partner-level allocations. Dorgham et al. (2022) extend collaborative network design to the healthcare context by proposing a collaborative hospital supply chain network design problem under uncertainty, illustrating that collaborative design and cost sharing are equally relevant in critical sectors such as hospital logistics. Their consideration of uncertainty and sector-specific constraints supports the idea that cooperative game models and allocation rules must be adaptable to different industries and robust under data variability. Mrabti et al. (2022) provide a comprehensive literature review on sustainable horizontal collaboration, synthesizing work on environmental, economic, and social dimensions of collaboration and emphasizing that cost allocation is a key enabler of long-term, sustainable partnerships. Their review shows that many studies propose collaborative structures but do not always specify rigorous allocation rules, thereby reinforcing the contribution of a systematic cooperative game framework that explicitly connects sustainability goals with incentive-compatible cost sharing. In another contribution, Mrabti et al. (2022) develop a multi-objective optimisation model for sustainable collaborative hub location and cost sharing, explicitly integrating cost allocation into the optimisation framework and showing that sustainability targets and equitable cost distribution can be addressed simultaneously. This supports the use of game-theoretic concepts in multi-objective settings to ensure that collaborative designs are both efficient and acceptable to participants. Varas et al. (2022) investigate a horizontal collaborative approach for planning wine grape harvesting, providing a sector-specific application where multiple producers coordinate harvesting and logistics decisions. Their results show that horizontal collaboration can yield significant operational improvements, but they also implicitly highlight the need for fair allocation of shared harvesting and transport costs to maintain producer commitment.

2. PROBLEM DESCRIPTION & NOTATION

2.1. Collaborative Logistics Scenario: Define:

(i) A set of **carriers / companies**: $N = \{1,2,\dots,N\}$. (1)

(ii) Each player has its own set of **customers and standalone routes.**

(iii) When they collaborate, they **pool routes/vehicles** to reduce total cost.

2.2. Notation: Introduce notation like:

N: set of players (carriers/shippers).

$S \subseteq N$: a coalition.

C_i: standalone cost of player iii.

$C(S)$: optimal collaborative cost for coalition SSS.

$v(S)$: characteristic function (worth) of coalition SSS.

x_i: allocated cost for player i.

c_{ij}: cost of traveling from node i to node j.

Decision variables of underlying routing model:

$$x_{ij}^{k} = \left\{\begin{matrix} 1 & \text{if vehicle k travels from i to j} \\ 0 & Otherwise \end{matrix}\right\} \tag{2}$$

Table 1: Notation and Description

Symbol	Description	Units
N	Set of collaborating carriers	–
C_i	Standalone cost of carrier (i)	₹ / €
$C(S)$	Collaborative cost of coalition (S)	₹ / €
$V(S)$	Coalition worth (savings)	₹ / €
x_i	Cost allocated to carrier (i)	₹ / €

3. METHODOLOGY

Cooperative Game Theory Framework:

3.1. Stepwise Framework:

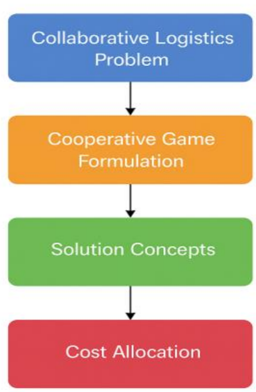

Figure 1: Proposed Cooperative Game Theory Framework for Collaborative Logistics

Figure (1) illustrates a stepwise framework for applying cooperative game theory to collaborative logistics. At the top, the collaborative logistics problem is identified, representing the practical context where multiple firms or carriers consider working together. This problem is then translated into a cooperative game formulation, where participants are modeled as players and their joint and individual costs or benefits are quantified. Next, solution concepts are applied, such as the Shapley value or nucleolus, to derive fair and theoretically sound ways of sharing gains or costs. Finally, the process leads to cost allocation, where the numerical results from the solution concepts are converted into specific cost shares assigned to each participant. The vertical flow with arrows highlights the logical progression from real-world logistics challenges to game-theoretic modeling and ultimately to implementable cost-sharing decisions.

3.2. Underlying Logistics Optimization Model: show how you compute C_i and $C(S)$ via a routing model (VRP).

Vehicle routing cost minimization

$$min \sum_{k\in K} \sum_{i\in V} \sum_{j\in V} c_{ij}x_{ij}^k \quad (3)$$

Subject to:

Flow conservation: $\sum_{j\in V} x_{ij}^k =: \sum_{j\in V} x_{ji}^k \ \ \forall\, i \in V\backslash\{0\}, k \in K$ (4)

Each customer served once:$\sum_{k\in K} \sum_{j\in V} x_{ij}^k = 1 \forall\, i \in \text{Customers}$ (5)

Vehicle capacity constraint, etc.

For each single player i, solve this model only for its customers → obtain standalone cost C_i.

For each coalition S, solve the integrated model for all customers in S → obtain collaborative cost $C(S)$.

3.3. Cooperative Game Construction: Define:

Characteristic function: $v(S) = \sum_{i\in S} C_i - C(S), \forall\, S \subseteq N, S \not\equiv \phi$ (6)

This represents **total savings** of coalition S compared to no collaboration.

Grand coalition worth: $v(N) = \sum_{i\in N} C_i - C(N)$ (7)

Cooperative game:(N, v) (8)

3.4. Cost Allocation Rules: Let $x = (x_1, x_2, \dots, x_n)$ be the **cost allocation** (each player's cost after collaboration). Equivalent to dividing total collaborative cost $C(N)$ or total savings $v(N)$.

Allocate **cost**: $\sum_{i\in N} x_i = C(N)$ (9)

Or allocate **savings**: $\sum_{i\in N} y_i = v(N)$ (10)

3.4.1. Shapley Value: First define value in terms of **savings**. The Shapley value of player i is:

$$\phi_i(v) = \sum_{S\subseteq N\backslash i} \frac{|S|!(|N|-|S|-1)!}{|N|!}[v(S \cup \{i\}) - v(S)] \quad (11)$$

If we allocate **savings**, the allocated saving is $\phi_i(v)$ and the **allocated cost** becomes:

$$x_i^{Shapley} = C_i - \phi_i(v) \quad (12)$$

Properties to highlight:

Efficiency: $\sum_i \phi_i(v) = v(N)$ (13)

Symmetry, dummy player, additivity.

3.4.2. Nucleolus: The nucleolus minimizes the dissatisfaction (excess) of coalitions. **Excess** of coalition S under allocation x (cost view):

$$e(S, x) = v(S) - (\sum_{i\in S} C_i - \sum_{i\in S} x_i) \quad (14)$$

The nucleolus is defined as the allocation that **lexicographically minimizes** the sorted vector of excesses over all coalitions.

3.4.3. Proportional Rule / Other Simple Rules: Allocate costs proportional to standalone costs:

$x_i^{Prop} = \frac{C_i}{\sum_{j\in N} C_j} C(N)$ Or proportional to demand/volume. (15)

3.5. Stability Analysis (Core, Individual Rationality): Define the core:

An allocation x (in cost space) is in the core if:

Efficiency: $\sum_{i\in N} x_i = C(N)$ (16)

Coalitional rationality: $\sum_{i\in S} x_i \le C(S), \forall\, S \subseteq N$ (17)

Individual rationality: $x_i \le C_i, \forall i \in N$ (18)

If allocation lies in the core, no coalition has incentive to deviate.

4. NUMERICAL ILLUSTRATION / CASE STUDY

4.1. Instance Description:

(i) Number of players (e.g., 3–5 carriers).

(ii) Demand sets, distances, cost parameters (fuel, labor).

(iii) Scenario: urban delivery, regional distribution, etc.

4.2. Standalone and Coalition Costs: Compute:

(i)Standalone costs $C_1, C_2, ..., C_n$

(ii) Collaborative costs $C(S)$ for relevant coalitions.

Table 2. Standalone and Coalition Costs

Coalition (S)	$\sum_{i\in S} C_i$	$C(S)$	$v(S) = \sum C_i - C(S)$
{1}	1000	1000	0
{2}	1200	1200	0
{1,2}	2200	1800	400
{1,2,3}	3300	2400	900
…	…	…	…

4.3. Cost Allocation Results: Calculate allocations via each method (Shapley, nucleolus, proportional).

Table 3. Cost Allocation under Different Rules

Player	Standalone cost C_i	Shapley cost x_i^{Sh}	Nucleolus cost x_i^{Nu}	Proportional cost x_i^{Pr}
1	1000	800	820	850
2	1200	900	880	950
3	1100	700	700	600
Total	3300	2400	2400	2400

4.4. Stability Check: For each allocation rule, evaluate:

(i) Efficiency: sum equals $C(N)$?

(ii)Individual rationality $x_i \leq C_i$

(iii)Core constraints $\sum_{i \in S} x_i = C(S)$

Table 4: Stability Indicators			
Rule	**Core Feasible?**	**All $x_i \leq C_i$?**	**Max excess $e(S, x)$**
Shapley	Yes	Yes	0
Nucleolus	Yes	Yes	0
Proportional	No	Some violations	150

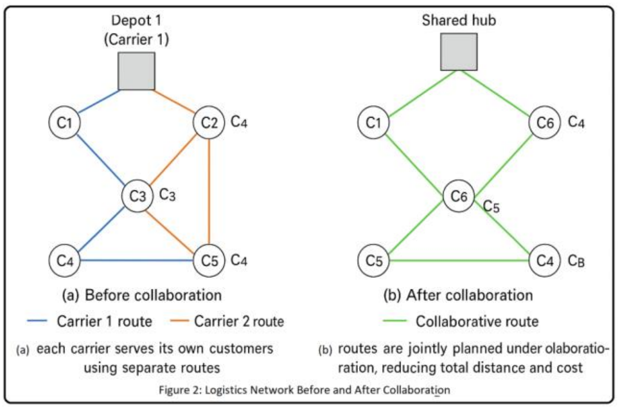

Figure 2: Logistics Network Before and After Collaboration

Figure (2) compares a logistics network before and after collaboration between carriers. In panel (a), labeled "Before collaboration," two separate routes are shown from a single depot representing Carrier 1. Customers are visited using distinct blue and orange paths, indicating that each carrier (or route) serves its own set of customers independently, leading to overlapping and potentially inefficient routes. In panel (b), labeled "After collaboration," a shared hub replaces the individual depot, and all customers are connected through a single green route. This illustrates a jointly planned collaborative route in which carriers pool their resources, serve customers together, and thereby reduce total travel distance and overall logistics cost.

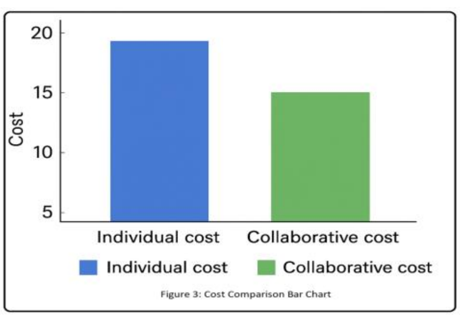

Figure 3: Cost Comparison Bar Chart

Figure (3) presents a cost comparison bar chart showing the difference between operating individually and under collaboration. The vertical axis represents cost, while the horizontal axis distinguishes between individual cost and collaborative cost. The blue bar, corresponding to individual cost, reaches a higher value of around 19 units, indicating that when firms operate separately their total logistics cost is relatively high. The green bar, representing collaborative cost, is shorter at about 15 units, illustrating that cooperation among participants leads to a noticeable reduction in overall cost. The chart visually emphasizes the economic benefit of collaboration in logistics operations.

5. MANAGERIAL INSIGHTS AND DISCUSSION

From a managerial perspective, the choice of cost allocation rule involves balancing fairness and stability, as illustrated by the numerical results and visual comparisons in Figures 2 and 3 and Tables 3 and 4. The Shapley value is attractive because it reflects each carrier's marginal contribution to the coalition and is therefore perceived as fair, and in many instances lies close to or inside the core; however, it can be computationally intensive when the number of participants or coalitions grows. The nucleolus, in contrast, is explicitly designed to minimize the maximum dissatisfaction of any coalition and thus provides the strongest stability guarantees, but it is even more complex to compute and explain to non-technical stakeholders. Simple proportional rules based on standalone costs or demand volumes are far easier to communicate and implement in contracts, yet they may lie outside the core, creating incentives for some firms to defect from collaboration, as seen when proportional allocations yield higher costs for certain carriers than standalone operation. Managers should therefore complement the static comparison of rules with sensitivity analysis, examining how changes in fuel prices, vehicle capacities, or demand patterns shift standalone and coalition costs and, consequently, the attractiveness of each rule. Translating any chosen allocation into enforceable agreements requires clear contractual mechanisms for side payments, revenue sharing, or penalty clauses that prevent opportunistic exit once savings materialize. Finally, to make the framework scalable to large collaborative networks, approximate methods such as sampling-based Shapley estimators or decomposition approaches for coalition cost evaluation may be needed, allowing firms to retain the fairness and stability benefits highlighted in the small-scale examples while keeping computational effort and negotiation complexity at manageable levels.

6. CONCLUDING REMARKS

The proposed cooperative game theory framework demonstrates that effective cost allocation is central to turning the theoretical benefits of collaborative logistics into sustainable practice. By linking an optimization-based representation of collaborative routing with game-theoretic solution concepts, the approach provides managers with a structured way to quantify coalition savings and to translate them into fair and stable cost shares. The numerical illustration shows that sophisticated rules such as the Shapley value and the nucleolus can keep all partners at least as well off as in standalone operation, while simple proportional rules, although more intuitive, may violate core constraints and create incentives to defect. These insights underline the importance of combining computational tools, sensitivity analysis and clear contractual arrangements, including side payments and revenue-sharing clauses, when implementing collaboration schemes. Future work can extend the framework to stochastic demand, multi-period planning and large-scale instances using approximation techniques, but the present study already offers a practical template for designing, evaluating and negotiating cost-sharing agreements in a wide range of collaborative logistics contexts.

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