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ON
ADVANCE AND APPLIED RESEARCH

PART-01

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PREFACE

We are pleased to present this edited book, which is a compilation of selected research papers from the International Conference on Advance and Applied Research. This conference serves as a platform for researchers, scholars, academicians, and industry experts from diverse disciplines to share their valuable insights and findings. The International Conference on Advance and Applied Research brings together intellectual minds from around the world to foster collaboration, exchange knowledge, and push the boundaries of research in various fields. The papers included in this book represent a culmination of rigorous research and innovative ideas that have the potential to shape the future of academia and industry.

The theme of the conference, "ADVANCE AND APPLIED RESEARCH," resonates throughout the chapters of this book. The selected papers cover a wide range of disciplines, including science, technology, engineering, social sciences, humanities, and management. Each chapter showcases the authors' expertise, dedication, and commitment to advancing knowledge in their respective fields. As editors, we would like to express our gratitude to the authors for their valuable contributions. Their hard work, perseverance, and passion for research are evident in the depth and quality of their papers. We would also like to acknowledge the rigorous review process that ensured the inclusion of only the most impactful and insightful research papers in this volume.

We extend our appreciation to the organizing committee, conference chairs, and session chairs for their unwavering support and guidance throughout the conference and the editing process. Their expertise and commitment were crucial in ensuring the success of the conference and the production of this edited book.

Furthermore, we would like to express our gratitude to the reviewers who generously volunteered their time and expertise to provide constructive feedback and enhance the quality of the papers. Their valuable input played a significant role in shaping the final versions of the chapters. We are grateful to the conference attendees, both in-person and virtual, who actively participated in the conference sessions, engaged in fruitful discussions, and contributed to the vibrant academic atmosphere. Your presence and enthusiasm enriched the conference experience for everyone involved.

Finally, we would like to thank the publisher for their support and assistance in bringing this edited book to fruition. Their professionalism and dedication to academic publishing have been instrumental in ensuring the timely publication of this valuable collection of research papers. It is our sincere hope that this edited book will serve as a valuable resource for researchers, scholars, and practitioners seeking to explore the latest advancements and trends in their respective fields. May the chapters within these pages inspire further research, stimulate critical thinking, and contribute to the ongoing pursuit of knowledge.

Thank you, and we hope that this edited book will be a catalyst for future research endeavours and collaborations.

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CHAPTER-45

A CLOSE APPROXIMATION OF THE SOLUTION TO THE HEAT EQUATION CONTAINING A DEPENDENT LINEAR CONVECTION FACTOR USING THE DOUBLE INTERPOLATION METHOD

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ABSTRACT

The heat equation is an example of a partial differential equation that can be used to describe the time-dependent concentration gradient across a domain of some quantity (like heat, for example). In this study, the dependent source term in the heat equation and its solution are the primary focuses of our attention. In the interest of transparency, we will assume that the source term is linear. Let's have a look at an example to better understand this topic. Due to the complexity of the underlying source terms, the answer cannot be reached by the process of variable separation. The double interpolation method is one of the novel approaches that we have investigated in order to solve the heat equation which has a dependent source term. By applying the formula for the recurrence relation in finite differences, we were able to determine the values of u(x,t) at each point on the lattice. Both the twofold interpolation method and the numerical answer have been examined side by side in this graphic comparison that we have created.

Keywords: *heat equation, convection term, double interpolation method*

INTRODUCTION

An example of a continuum that can be described by the heat equation is a solid, liquid, or gas. The heat equation is a form of partial differential equation used to describe how heat moves through these media. This provides the mathematical foundation for understanding what thermal conductivity really is. In other words, the heat equation explains how heat (or temperature change) propagates through a region over time. Finding a solution to the one-dimensional heat equation is extremely important due to its popularity in the fields of science and engineering. In addition, the topic of thermal equation simulation methods has emerged as a particularly interesting subfield in the field of digital solutions. This is one of the reasons why this sport is so popular. Numerical simulation of heat flows is a difficult topic to be researched and developed by many different scientific institutions.

In contrast to the more common separation of variables approach, Adomian's method for solving thermal equations was evaluated by Gorguis and Chan (2008).

The homotopy perturbation method was compared with other methods for solving two-point boundary value problems in a 2010 paper by Chun et al. The homotopy perturbation method was used to solve two-point linear and nonlinear boundary value problems and was compared with the extended Adomian decomposition and snapping methods. Hemeda (2012) considered the homotopy perturbation technique to solve systems of nonlinear coupled equations. He used HPM to solve many partial differential equations, including one- and two-dimensional systems of the Burgers limit equation and the Laplace system of equations. The survey results support the claims of HPM's superiority over other digital methods in terms of power, user-friendliness, and efficiency. Kim & Lee (2013) discussed a new numerical method for solving hyperbolic equations with rapidly changing components. Using a solution-interpolation-based approach, they construct a numerical diagram by combining numerical data with a parametric solution of the equation. JuN et al. (2015) propose a method of local homotopy perturbation as a way to go beyond the standard homotopy perturbation strategy. Two examples were used to study the efficiency and convergence of the local fractional homotopy perturbation method. To solve the diffusion and heat equations in a new approach, Yang and Gao (2017) proposed an iterative transform and an integral transformation similar to the Sumudu transform. The approach is said to provide very accurate and efficient approximation solutions to partial differential equations. Nolasco et al. (2019) provided an experimental model of cooling effects to evaluate the influence of key physical parameters and eventually resulted in an analytical heat transfer model. The study authors solved an analytic model of heat transfer in pot-cooker refrigerators using explicit and implicit numerical methods. Babaei et al. (2020) solved the numerical solutions for multidimensional integral differential equations of variable order. Rozin Khatun and Shajib Ali published a study in 2020 analyzing the robustness of one-dimensional thermal equations. The thermal equation has been analyzed as an infinite-dimensional initial value problem. They found that while centralized diff schemes (CNS) worked well for this problem, focused explicit diff schemes (ECDS) did not. The thermal equation for the size of the polar cylinder has been numerically solved by Hussain et al. (2021) using the non-mesh line method. The spatial variables are discretized with a multi-order radial basis function and time integration is performed by the Runge-Kutta method of order 4. Similar to the explicit central difference method, Loskor and Sarkar (2022) proposed a numerical solution using the finite difference method. Forward Time and Space Centered (FTCS) was used to solve the problem related to the one-dimensional thermal equation and the stability conditions of the scheme were reported for different thermal conductivity of the materials.

LINEAR CONVECTION HEAT EQUATION WITH A DEPENDENT VARIABLE

Again, a separation of variables might be used to obtain solutions for the heat equation with a solution-dependent linear convection term, but this approach would fail in the case of more complex convection terms. So, we take another look at the problem, this time employing the double interpolation technique.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \delta \frac{\partial u}{\partial x}, 0 < x < 1, t > 0 \tag{1}$$

Initial and boundary condition is given as

$$u(x,0) = \mathrm{x} - \mathrm{x}^2 , u(0,t) = 0, u(1,t) = 0 \tag{2}$$

where δ is some constant.

ANALYTIC SOLUTION OF THE PROPOSED HEAT EQUATION

The solutions for

$\delta = 6$

$$u(x,t) = -4\pi e^{-3x-9t}\left[-\frac{27+\pi^2+2\pi^2 e^3}{(9+\pi^2)^3}e^{-\pi^2 t}sin\pi x + \frac{54+8\pi^2+16\pi^2 e^3}{(9+4\pi^2)^3}e^{-4\pi^2 t}sin2\pi x + \cdots\right]$$

$\delta = -12$

$$u(x,t) = 2\pi e^{6x-36t}\left[-\frac{108+7\pi^2+(5\pi^2+324)e^{-6}}{(36+n^2\pi^2)^3}e^{-\pi^2 t}sin\pi x + \frac{206+56\pi^2+(40\pi^2+648)e^{-6}}{(36+4\pi^2)^3}e^{-4\pi^2 t}sin2\pi x + \cdots\right]$$

APPROXIMATE SOLUTION BY DOUBLE INTERPOLATION METHOD

Now we will solve the equation (1) along with conditions (2) double interpolation method, we get

Difference of x by a factor of 0.2, or $h = 0.2$

$$k = \frac{h^2}{2} = \frac{(0.2)^2}{2} = 0.02$$

From finite difference, the time span of t as

$$\lambda = \frac{k}{h^2} = \frac{0.02}{(0.2)^2} = \frac{0.02}{0.04} = \frac{1}{2} = 0.5$$

$$\mu = \frac{k}{h} = \frac{0.02}{0.2} = 0.1$$

Let us take $\delta = 6$

$$u_{i,j+1} = (1 - 2\lambda - 6\mu)u_{i,j} + (\lambda + 6\mu)u_{i+1,j} + \lambda u_{i-1,j}$$

$$u_{i,j+1} = -0.6u_{i,j} + 1.1u_{i+1,j} + 0.5u_{i-1,j} \qquad (3)$$

Thus $x_0 = 0, x_1 = 0.2, x_2 = 0.4, x_3 = 0.6, x_4 = 0.8, x_5 = 1$

$t_0 = 0, t_1 = 0.02, t_2 = 0.04, t_3 = 0.06, t_4 = 0.08, t_5 = 0.1$

After connecting all the dots with lines that run perpendicular to the x -axis and the t -axis, we obtain a 25-point mesh.

$$u_{i,j+1} = -0.6u_{i,j} + 1.1u_{i+1,j} + 0.5u_{i-1,j} \qquad (4)$$

$u_{11} = -0.6u_{10} + 1.1u_{20} + 0.5u_{00} = 0.1680$; $u_{21} = -0.6u_{20} + 1.1u_{30} + 0.5u_{10} = 0.3296$

$u_{12} = -0.6u_{11} + 1.1u_{21} + 0.5u_{01} = 0.2618$; $u_{31} = -0.6u_{30} + 1.1u_{40} + 0.5u_{20} = 0.2816$

$u_{41} = -0.6u_{40} + 1.1u_{50} + 0.5u_{30} = 0.0240$; ; $u_{22} = -0.6u_{21} + 1.1u_{31} + 0.5u_{11} = 0.1960$

$u_{32} = -0.6u_{31} + 1.1u_{41} + 0.5u_{21} = 0.0222$; $u_{42} = -0.6u_{41} + 1.1u_{51} + 0.5u_{31} = 0.1264$

$u_{13} = -0.6u_{12} + 1.1u_{22} + 0.5u_{02} = 0.0585$; $u_{23} = -0.6u_{22} + 1.1u_{32} + 0.5u_{12} = 0.0377$

$u_{33} = -0.6u_{32} + 1.1u_{42} + 0.5u_{22} = 0.2237$; $u_{43} = -0.6u_{42} + 1.1u_{52} + 0.5u_{32} = -0.0647$

$u_{14} = -0.6u_{13} + 1.1u_{23} + 0.5u_{03} = 0.0064$; $u_{24} = -0.6u_{23} + 1.1u_{33} + 0.5u_{13} = 0.0311$

$u_{34} = -0.6u_{33} + 1.1u_{43} + 0.5u_{23} = -0.1865$; $u_{44} = -0.6u_{43} + 1.1u_{53} + 0.5u_{33} = 0.1507$

$u_{15} = -0.6u_{14} + 1.1u_{24} + 0.5u_{04} = 0.0304$; $u_{25} = -0.6u_{24} + 1.1u_{34} + 0.5u_{14} = -0.2206$

$u_{35} = -0.6u_{34} + 1.1u_{44} + 0.5u_{24} = 0.2932$; $u_{45} = -0.6u_{34} + 1.1u_{44} + 0.5u_{24} = 0.2932$

Table-1

	0	0.2	0.4	0.6	0.8	1
0	0	0.16	0.24	0.24	0.16	0
0.02	0	0.1680	0.3296	0.2816	0.0240	0
0.04	0	0.2618	0.1960	0.0222	0.1264	0
0.06	0	0.0585	0.0377	0.2237	-0.0647	0
0.08	0	0.0064	0.0311	-0.1865	0.1507	0
0.1	0	0.0304	-0.2206	0.2932	0.2932	0

Table-2

S.No.	u_{1i}	$\Delta^{0+1}u_{1i}$	$\Delta^{0+2}u_{1i}$	$\Delta^{0+3}u_{1i}$	$\Delta^{0+4}u_{1i}$	$\Delta^{0+5}u_{1i}$
1	0.16	0.008	0.0858	-0.3829	0.8312	-1.3546
2	0.168	0.0938	-0.2971	0.4483	-0.5234	
3	0.2618	-0.2033	0.1512	-0.0751		
4	0.0585	-0.0521	0.0761			
5	0.0064	0.024				
6	0.0304					

Table-3

S.No.	u_{2i}	$\Delta^{0+1}u_{2i}$	$\Delta^{0+2}u_{2i}$	$\Delta^{0+3}u_{2i}$	$\Delta^{0+4}u_{2i}$	$\Delta^{0+5}u_{2i}$
1	0.24	0.0896	-0.2232	0.1985	-0.0221	-0.5511
2	0.3296	-0.1336	-0.0247	0.1764	-0.5732	
3	0.196	-0.1583	0.1517	-0.3968		
4	0.0377	-0.0066	-0.2451			
5	0.0311	-0.2517				
6	-0.2206					

Table-4

S.No.	u_{3i}	$\Delta^{0+1}u_{3i}$	$\Delta^{0+2}u_{3i}$	$\Delta^{0+3}u_{3i}$	$\Delta^{0+4}u_{3i}$	$\Delta^{0+5}u_{3i}$
1	0.24	0.0416	-0.301	0.7619	-1.8345	4.4087
2	0.2816	-0.2594	0.4609	-1.0726	2.5742	

4	0.2237	-0.4102	0.8899			
5	-0.1865	0.4797				
6	0.2932					

Table-5

S.No.	u_{4i}	$\Delta^{0+1}u_{4i}$	$\Delta^{0+2}u_{4i}$	$\Delta^{0+3}u_{4i}$	$\Delta^{0+4}u_{4i}$	$\Delta^{0+5}u_{4i}$
1	0.16	-0.136	0.2384	-0.5319	1.2319	-2.4113
2	0.024	0.1024	-0.2935	0.7	-1.1794	
3	0.1264	-0.1911	0.4065	-0.4794		
4	-0.0647	0.2154	-0.0729			
5	0.1507	0.1425				
6	0.2932					

Table-6

S.No.	u_{i0}	$\Delta^{1+0}u_{i0}$	$\Delta^{2+0}u_{i0}$	$\Delta^{3+0}u_{i0}$	$\Delta^{4+0}u_{i0}$	$\Delta^{5+0}u_{i0}$
1	0	0.16	-0.08	0	0	0
2	0.16	0.08	-0.08	0	0	
3	0.24	0	-0.08	0		
4	0.24	-0.08	-0.08			
5	0.16	-0.16				
6	0					

Table-7

S.No.	u_{i1}	$\Delta^{1+0}u_{i1}$	$\Delta^{2+0}u_{i1}$	$\Delta^{3+0}u_{i1}$	$\Delta^{4+0}u_{i1}$	$\Delta^{5+0}u_{i1}$
1	0	0.168	-0.0064	-0.2032	0.2032	0.24
2	0.168	0.1616	-0.2096	0	0.4432	
3	0.3296	-0.048	-0.2096	0.4432		
4	0.2816	-0.2576	0.2336			
5	0.024	-0.024				
6	0					

Table-8

S.No.	u_{i2}	$\Delta^{1+0}u_{i2}$	$\Delta^{2+0}u_{i2}$	$\Delta^{3+0}u_{i2}$	$\Delta^{4+0}u_{i2}$	$\Delta^{5+0}u_{i2}$
1	0	0.2618	-0.3276	0.2196	0.1664	-1.061
2	0.2618	-0.0658	-0.108	0.386	-0.8946	
3	0.196	-0.1738	0.278	-0.5086		

4	0.0222	0.1042	-0.2306			
5	0.1264	-0.1264				
6	0					

Table-9

S.No.	u_{i3}	$\Delta^{1+0}u_{i3}$	$\Delta^{2+0}u_{i3}$	$\Delta^{3+0}u_{i3}$	$\Delta^{4+0}u_{i3}$	$\Delta^{5+0}u_{i3}$
1	0	0.0585	-0.0793	0.2861	-0.9673	2.476
2	0.0585	-0.0208	0.2068	-0.6812	1.5087	
3	0.0377	0.186	-0.4744	0.8275		
4	0.2237	-0.2884	0.3531			
5	-0.0647	0.0647				
6	0					

Table-10

S.No.	u_{i4}	$\Delta^{1+0}u_{i4}$	$\Delta^{2+0}u_{i4}$	$\Delta^{3+0}u_{i4}$	$\Delta^{4+0}u_{i4}$	$\Delta^{5+0}u_{i4}$
1	0	0.0064	0.0183	-0.2606	1.0577	-2.8975
2	0.0064	0.0247	-0.2423	0.7971	-1.8398	
3	0.0311	-0.2176	0.5548	-1.0427		
4	-0.1865	0.3372	-0.4879			
5	0.1507	-0.1507				
6	0					

Table-11

S.No.	u_{i5}	$\Delta^{1+0}u_{i5}$	$\Delta^{2+0}u_{i5}$	$\Delta^{3+0}u_{i5}$	$\Delta^{4+0}u_{i5}$	$\Delta^{5+0}u_{i5}$
1	0	0.0304	-0.2814	1.0462	-2.3248	3.824
2	0.0304	-0.251	0.7648	-1.2786	1.4992	
3	-0.2206	0.5138	-0.5138	0.2206		
4	0.2932	0	-0.2932			
5	0.2932	-0.2932				
6	0					

Table 1's first and last columns both include zeros, hence it follows that

$$\Delta^{0+1}u_{00} = \Delta^{0+2}u_{00} = \Delta^{0+3}u_{00} = \Delta^{0+4}u_{00} = \Delta^{0+5}u_{00} = 0 \qquad (5)$$

And $\Delta^{0+1}u_{50} = \Delta^{0+2}u_{50} = \Delta^{0+3}u_{50} = \Delta^{0+4}u_{50} = \Delta^{0+5}u_{50} = 0$ (6)

From Table 2, we get

$$\Delta^{0+1}u_{10} = 0.008, \Delta^{0+2}u_{10} = 0.0858, \Delta^{0+3}u_{10} = -0.3829, \Delta^{0+4}u_{10} = 0.8312, \Delta^{0+5}u_{10} = -1.3546 \qquad (7)$$

From Table 3

$$\Delta^{0+1}u_{20} = 0.0896, \Delta^{0+2}u_{20} = -0.2232, \Delta^{0+3}u_{20} = 0.1985, \Delta^{0+4}u_{20} = -0.0221, \Delta^{0+5}u_{20} = -0.5511 \quad (8)$$

From Table 4

$$\Delta^{0+1}u_{30} = 0.0416, \Delta^{0+2}u_{30} = -0.301, \Delta^{0+3}u_{30} = 0.7619, \Delta^{0+4}u_{30} = -1.8345, \Delta^{0+5}u_{30} = 4.4087 \quad (9)$$

From Table 5

$$\Delta^{0+1}u_{40} = -0.136, \Delta^{0+2}u_{40} = 0.2384, \Delta^{0+3}u_{40} = -0.5319, \Delta^{0+4}u_{40} = 1.2319, \Delta^{0+5}u_{40} = -2.4113 \quad (10)$$

From Table 6

$$\Delta^{1+0}u_{00} = 0.16, \Delta^{2+0}u_{00} = -0.08, \Delta^{3+0}u_{00} = 0, \Delta^{4+0}u_{00} = 0, \Delta^{5+0}u_{00} = 0 \quad (11)$$

From Table 7

$$\Delta^{1+0}u_{01} = 0.168, \Delta^{2+0}u_{01} = -0.0064, \Delta^{3+0}u_{01} = -0.2032, \Delta^{4+0}u_{01} = 0.2032, \Delta^{5+0}u_{01} = 0.24 \quad (12)$$

From Table 8

$$\Delta^{1+0}u_{02} = 0.2618, \Delta^{2+0}u_{02} = -0.3276, \Delta^{3+0}u_{02} = 0.2196, \Delta^{4+0}u_{02} = 0.1664, \Delta^{5+0}u_{02} = -1.061 \quad (13)$$

From Table 9

$$\Delta^{1+0}u_{03} = 0.0585, \Delta^{2+0}u_{03} = -0.0793, \Delta^{3+0}u_{03} = 0.2861, \Delta^{4+0}u_{03} = -0.9673, \Delta^{5+0}u_{03} = 2.476 \quad (14)$$

From Table 10

$$\Delta^{1+0}u_{04} = 0.0064, \Delta^{2+0}u_{04} = 0.0183, \Delta^{3+0}u_{04} = -0.2606, \Delta^{4+0}u_{04} = 1.0577, \Delta^{5+0}u_{04} = -2.8975 \quad (15)$$

From Table 11

$$\Delta^{1+0}u_{05} = 0.0304, \Delta^{2+0}u_{05} = -0.2814, \Delta^{3+0}u_{05} = 1.0462, \Delta^{4+0}u_{05} = -2.3248, \Delta^{5+0}u_{05} = 3.824 \quad (16)$$

The general expression for finding the gaps between two orders is

$$\Delta^{m+n}u_{00} = \Delta^{m+0}u_{0n} - n\Delta^{m+0}u_{0n-1} + \frac{n(n-1)}{2!}\Delta^{m+0}u_{0n-2} - \cdots + (-1)^m\Delta^{m+0}u_{00} \quad (17)$$

$$\Delta^{n+m}u_{00} = \Delta^{0+n}u_{m0} - m\Delta^{0+n}u_{m-10} + \frac{m(m-1)}{2!}\Delta^{0+n}u_{m-20} - \cdots + (-1)^m\Delta^{0+n}u_{00} \quad (18)$$

$$\Delta^{1+1}u_{00} = \Delta^{1+0}u_{01} - \Delta^{1+0}u_{00} = 0.0080 \quad (19)$$

$$\Delta^{1+2}u_{00} = \Delta^{1+0}u_{02} - 2\Delta^{1+0}u_{01} + \Delta^{1+0}u_{00} = 0.0858 \quad (20)$$

$$\Delta^{2+1}u_{00} = \Delta^{2+0}u_{01} - \Delta^{2+0}u_{00} = 0.0736 \quad (21)$$

$$\Delta^{3+1}u_{00} = \Delta^{3+0}u_{01} - \Delta^{3+0}u_{00} = -0.2032 \quad (22)$$

$$\Delta^{1+3}u_{00} = \Delta^{1+0}u_{03} - 3\Delta^{1+0}u_{02} + 3\Delta^{1+0}u_{01} - \Delta^{1+0}u_{00} = -0.3829 \quad (23)$$

$$\Delta^{2+2}u_{00} = \Delta^{2+0}u_{02} - 2\Delta^{2+0}u_{01} + \Delta^{2+0}u_{00} = -0.3948 \quad (24)$$

$$\Delta^{1+4}u_{00} = \Delta^{1+0}u_{04} - 4\Delta^{1+0}u_{03} + 6\Delta^{1+0}u_{02} - 4\Delta^{1+0}u_{01} + \Delta^{1+0}u_{00} = 0.8312 \quad (25)$$

$$\Delta^{4+1}u_{00} = \Delta^{4+0}u_{01} - \Delta^{4+0}u_{00} = 0.2032 \quad (26)$$

$$\Delta^{3+2}u_{00} = \Delta^{3+0}u_{02} - 2\Delta^{3+0}u_{01} + \Delta^{3+0}u_{00} = \quad 0.6260 \tag{27}$$

$$\Delta^{2+3}u_{00} = \Delta^{2+0}u_{03} - 3\Delta^{2+0}u_{02} + 3\Delta^{2+0}u_{01} - \Delta^{2+0}u_{00} = 0.9643 \tag{28}$$

The following formula is used to interpolate polynomials in two variables up to the difference of the fifth degree.

$$u(x,t) =$$

$$u_{00} + \left[\frac{(x-x_0)}{h}\Delta^{1+0}u_{00} + \frac{(t-t_0)}{k}\Delta^{0+1}u_{00}\right]$$

$$+\frac{1}{2!}\left[\frac{(x-x_0)(x-x_1)}{h^2}\Delta^{2+0}u_{00} + \frac{2(x-x_0)(t-t_0)}{hk}\Delta^{1+1}u_{00} + \frac{(t-t_0)(t-t_1)}{k^2}\Delta^{0+2}u_{00}\right]$$

$$+\frac{1}{3!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)}{h^3}\Delta^{3+0}u_{00} + \frac{3(x-x_0)(x-x_1)(t-t_0)}{h^2k}\Delta^{2+1}u_{00} + \frac{3(x-x_0)(t-t_0)(t-t_1)}{hk^2}\Delta^{1+2}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)}{k^3}\Delta^{0+3}u_{00}\right]$$

$$+\frac{1}{4!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{h^4}\Delta^{4+0}u_{00} + \frac{4(x-x_0)(x-x_1)(x-x_2)(t-t_0)}{h^3k}\Delta^{3+1}u_{00} + \frac{6(x-x_0)(x-x_1)(t-t_0)(t-t_1)}{h^2k^2}\Delta^{2+2}u_{00} + \frac{4(x-x_0)(t-t_0)(t-t_1)(t-t_2)}{hk^3}\Delta^{1+3}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{k^4}\Delta^{0+4}u_{00}\right]$$

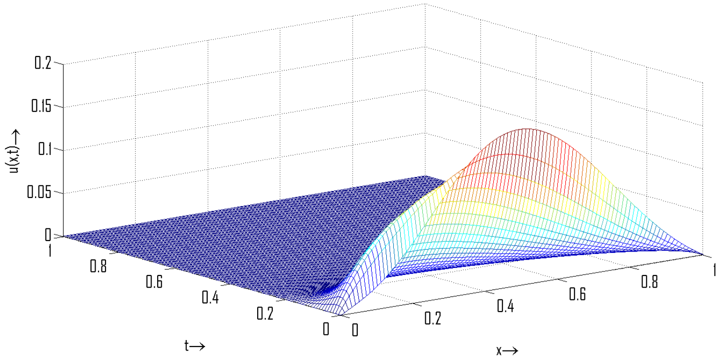
$$+\frac{1}{5!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{h^5}(0) + \frac{5(x-x_0)(x-x_1)(x-x_2)(x-x_3)(t-t_0)}{h^4k}\Delta^{4+1}u_{00} + \frac{10(x-x_0)(x-x_1)((x-x_2)(t-t_0)(t-t_1)}{h^3k^2}\Delta^{3+2}u_{00} + \frac{10(x-x_0)(x-x_1)(t-t_0)(t-t_1)(t-t_2)}{h^2k^3}\Delta^{2+3}u_{00} + \frac{5(x-x_0)(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{hk^4}\Delta^{1+4}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{k^5}\Delta^{0+5}u_{00}\right] \tag{29}$$

$$u(x,t) =$$

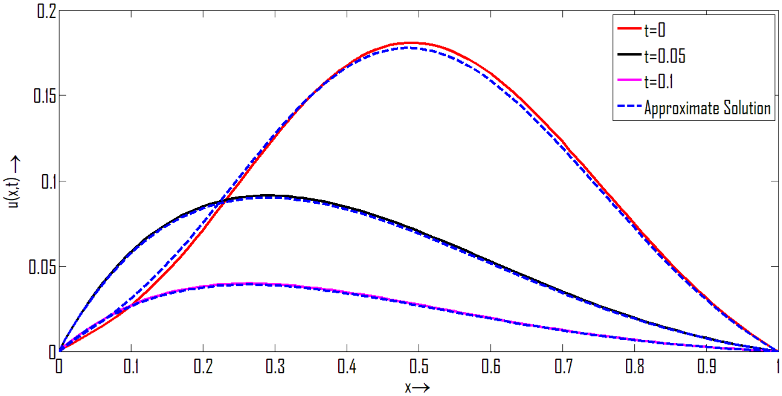
$$0 + 0.8x + (0.2x - x^2 + 2xt) + [92xt(x-0.2) + 1072.5xt(t-0.02)] - [211.667x(x-0.2)(x-0.4)(t-0.02) + 6168.8x(x-0.2)(t-0.02)(t-0.04) + 39885xt(t-0.02)(t-0.04)] + [31750xt(x-0.2)(x-0.4)(x-0.6) + 1956200x(x-0.2)((x-0.4)(t-0)(t-0.04) + 30134000x(x-0.2)(t-0)(t-0.02)(t-0.04) + 15067000x(x-0.2)(t-0)(t-0.02)(t-0.04) + 129880000(x-0.2)(t-0)(t-0.02)(t-0.04)(t-0.06)] + \cdots \tag{30}$$

RESULT AND DISCUSSION

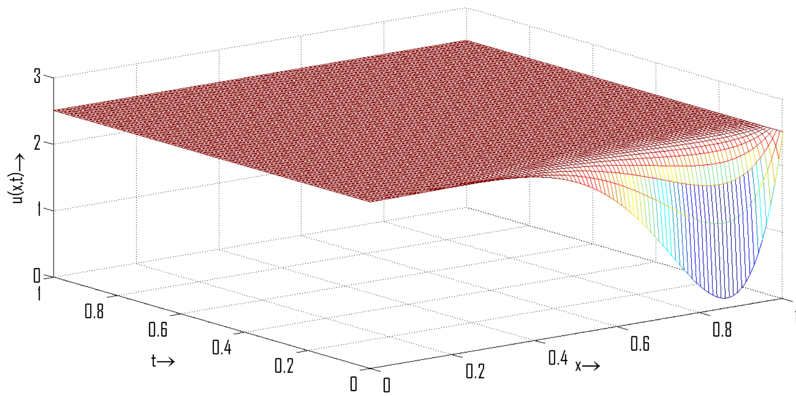
Graph 1: 3D Solution of proposed heat equation by double interpolation method δ=6



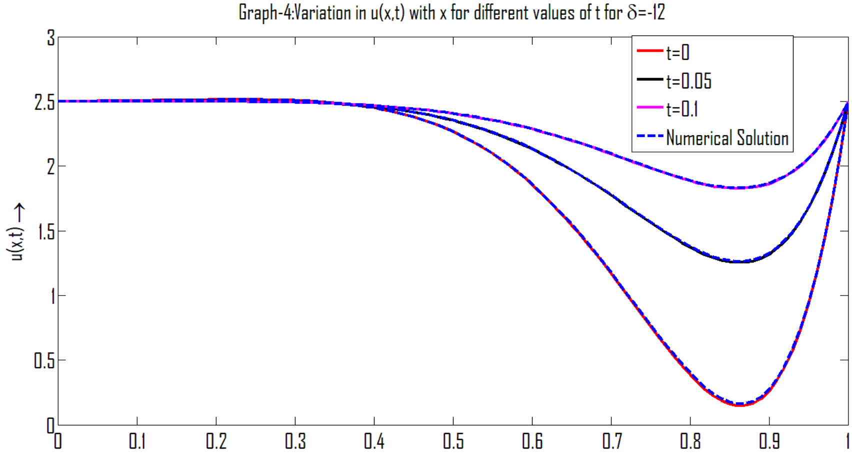
Graph-2:Variation in u(x,t) with x for different values of t and δ=6



Graph 3: 3D Solution of proposed heat equation by double interpolation method for δ=-12



Graph-4:Variation in u(x,t) with x for different values of t for δ=-12



The space-time graph of the numerical solution to the suggested problem using the double interpolation approach is presented in graph (1). The value of the parameter δ equals 6. The graph (2) presents a comparison of the numerical and analytical solutions with x for a variety of t values, where $\delta = 6$ is the same constant value. The space-time graph of the numerical solution to the suggested issue using the double interpolation approach is presented in graph (3). The value of $\delta = -12$ was used in this example. In graph (4), we see a comparison of the numerical solution with x and the analytic solution with x for various values of t and $\delta = -12$. This comparison was made possible by the use of graphing software.

CONCLUDING REMARKS

The well-known one-dimensional solution to the heat equation with a dependent source term is approximated numerically using the double interpolation method presented in this study. The proposed method already guarantees the required precision with a modest number of grid points. Since the boundary conditions are automatically considered, the method is particularly practical for addressing initial boundary value issues. Numerical findings reveal that the suggested method is both effective for the numerical solution of the aforementioned issue and generalizable to other partial differential equations, and its implementation is quite straightforward.

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